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QUANTUM MACHINE LEARNING AND HYBRID QUANTUM-CLASSICAL OPTIMIZATION FOR COMPLEX SCHEDULING, COMBINATORIAL OPTIMIZATION, AND HIGH-DIMENSIONAL DATA ANALYSIS

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Abstract

The rapid expansion of supply chains, logistics networks and enterprise-scale manufacturing platforms has made computational problems harder. The problems involve NP-hard optimization and high-dimensional data processing. Classical exact solvers are slow on non-convex, constraint-dense search spaces. Metaheuristics such as Simulated Annealing and Genetic Algorithms often stop at local minima. Quantum computing offers an approach by using superposition, entanglement and quantum tunnelling. Nisq-era hardware limits the use of fault-tolerant, deep-circuit algorithms at production scale. This review looks at quantum-classical optimization and Quantum Machine Learning (QML) frameworks for complex scheduling, combinatorial optimization and high-dimensional analytics. It covers models like Quadratic Unconstrained Binary Optimization (QUBO) Ising Hamiltonian mappings, Quantum Annealing and the Quantum Approximate Optimization Algorithm (QAOA) as well as higher-order mappings and penalty tuning strategies. Special focus is on trainability landscapes studying the Barren Plateau through Dynamical Lie Algebra theory and reviewing ways to help such as Hamiltonian Variational Ansätze, logarithmic-depth circuits and dynamic parameterised circuits. Industrial workflows such as Job Shop Scheduling, Capacitated Vehicle Routing, AGV dispatching and satellite acquisition scheduling are evaluated with tests against Gurobi, Simulated Annealing, Parallel Tempering and DMRG tensor networks. QML algorithms like Quantum Support Vector Machines, Quantum Support Vector Data Description, Quantum Convolutional Neural Networks and Quantum Principal Component Analysis are examined through the dequantisation framework showing the limits where true quantum advantage appears. Results show that hybrid architectures can reach approximation ratios close, to 0.99 on medium-scale Max-Cut instances while dequantisation shows that many claimed speedups actually become polynomial under the same data-access assumptions meaning real advantage only stays for high-rank matrices and data that is truly quantum.

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1. INTRODUCTION

The ongoing growth of industrial supply chains, frequent movement of goods through logistics networks and large-scale manufacturing systems has made it harder to handle complex math problems and big data (Patra et al. 2026). Tasks like scheduling jobs in a factory, planning routes for delivery vehicles organizing satellite observations and managing robot movement become much harder as more factors are added. Traditional exact math tools, like ones that use integer linear programming in programs such as Gurobi take long when dealing with complicated and crowded problems (Lamichhane & Rawat 2025). Because of this companies often use methods like Simulated Annealing, Genetic Algorithms and Tabu Search. These methods give up on finding the best solution in favor of getting a good enough solution faster but they often end up with solutions that are not the best possible (Fazlioglu, 2026).

Quantum computing offers a way to solve problems by using the basic rules of quantum physics like superposition, entanglement and tunneling (Thakur et al., 2025). In the time of noisy quantum computers there are not enough real qubits, the time they stay in a state is short the way they change states is not perfect and the connections between them are limited. This makes it hard to run error-free quantum programs like Shor's or Grover's algorithms at a large scale (Vani, 2026). To fix this a new approach called quantum-classical computing has become popular. In these systems the part of the problem that's hard to solve is done on a quantum computer while the part that is easier is done on a regular supercomputer (Tera et al. 2025).

At the time quantum machine learning tools like Quantum Support Vector Machines, Quantum Support Vector Data Description and Quantum Principal Component Analysis have been created to handle big data sets much faster than regular computers (Acampora et al. 2025).. Proving that a quantum computer is faster than a regular one is still hard. Some new methods that work on computers have shown that many of the speedups from quantum methods come from how data is loaded like using a special memory system not from the actual quantum processing (Nassir et al. 2025). Also the methods used in both optimization and quantum machine learning face a problem called the Barren Plateau. This happens when the math gets harder and harder as more parts are added making it hard to find the solution (Madhavi et al. 2026).

This article gives a look at the hybrid quantum-classical methods used for complex scheduling, hard math problems and dealing with big data (Keçeci, 2025). It looks at the math, behind these methods the ways to build them how the math problems change as they get bigger how to use hybrid systems how they compare to traditional methods and the limits that show when a quantum computer is really faster (Raseena, 2025).

2. Mathematical Formulations and Computational Paradigms

2.1 Quadratic Unconstrained Binary Optimization (QUBO) and Higher-Order Mappings

To process combinatorial optimization problems on specialized quantum hardware such as Ising-model quantum annealers or variational gate-based quantum processors classical constrained optimization problems must be translated into discrete quadratic forms (Fu et al., 2025). The primary mathematical substrate for quantum optimization is the Quadratic Unconstrained Binary Optimization (QUBO) problem, defined as:

$$\min_{\mathbf{x}\in\{0,1\}^n} E(\mathbf{x}) = \mathbf{x}^T\mathbf{Q}\mathbf{x} = \sum_{i=1}^{n} Q_{ii}\, x_i + \sum_{i<j} Q_{ij}\, x_i x_j$$

where $\mathbf{x}$ is an n-dimensional vector of binary decision variables, and $\mathbf{Q} \in \mathbb{R}^{n\times n}$ is a symmetric or upper-triangular real matrix encoding both linear cost coefficients along its diagonal (Q_{ii}) and pairwise variable couplings in its off-diagonal elements (Q_{ij}) (Li et al., 2025).

To map a QUBO problem onto a physical quantum processing topology, binary variables $x_i \in \{0,1\}$ are transformed into spin variables $z_i \in \{-1,+1\}$ via the linear affine mapping $x_i = \frac{1-z_i}{2}$.

Substituting this mapping converts the objective function into an equivalent physical spin-glass Hamiltonian, known as the Ising Hamiltonian:

$$H_C = -\sum_{i<j} J_{ij}\,\sigma_i^z\sigma_j^z - \sum_{i=1}^{n} h_i\,\sigma_i^z + C_{\text{offset}}$$

where σ_i^z represents the Pauli-Z operator acting on qubit i, $J_{ij} = \frac{Q_{ij}}{4}$ denotes the inter-qubit coupling strength, $h_i = \frac{Q_{ii}}{2} + \sum_{j>i}\frac{Q_{ij}}{4} + \sum_{j<i}\frac{Q_{ji}}{4}$ represents the local magnetic field strength, and C_{offset} is an unobservable energy shift constant (Mandal et al., 2020).

Industrial operational constraints (e.g., resource allocations, single-assignment requirements, or precedence dependencies) are integrated into unconstrained QUBO matrices by introducing quadratic penalty functions. Given an equality constraint $\sum_i a_i\,x_i = b$, a squared penalty term $P\cdot(\sum_i a_i\,x_i - b)^2$ is added to the unconstrained cost function, where $P > 0$ is a penalty parameter (Uotila, 2025).

If P is too small when using the quantum solver it creates unfeasible solutions HispanicOdeiro, P is too large or not set high enough this overemphasis of penalty terms creates an obscuring of the cost function topology, which creates high energy steps and forces both the classical and quantum optimizers to spawn in local minima (Angara, 2025). Moreover, non-quadratic constraints or multi-variable interactions in a hard QUBO become HOBO. To convert HOBO into standard quadratic QUBO using order-reduction binarization schemes we need to add additional used variables (ancilla qubits) which leads to a superlinear variable blow-up ("bloating") that strains hardware resources (Bouchmal, 2025).

2.2 Quantum Annealing, QAOA, and Variational Frameworks

Quantum optimization strategies broadly split into analog continuous-time processes (Quantum Annealing) and digital gate-based discrete variational circuits (QAOA and VQE) (Zhou et al., 2020).

Quantum Annealing relies on the adiabatic theorem of quantum mechanics. The system is initialized in the known ground state of a simple, non-commuting driver Hamiltonian, typically $H_0 = -\sum_{i=1}^{n}\sigma_i^x$ (Volpe et al., 2025). The system evolves dynamically according to a time-dependent transverse-field Hamiltonian:

$$H(t) = \left(1 - s\left(\frac{t}{\tau}\right)\right)H_0 + s\left(\frac{t}{\tau}\right)H_C$$

where $s(t/\tau) \in [0,1]$ represents the annealing schedule over operational run duration τ. Provided the evolution rate $\frac{ds}{dt}$ is slow relative to the inverse square of the minimum spectral energy gap $\Delta_{\min} = \min_t(E_1(t) - E_0(t))$ between the ground state and the first excited state, the system remains in the instantaneous ground state, arriving at the global minimum of H_C at $t = \tau$ (Mbeng et al., 2019).

Simulated Annealing (SA) overcomes energy barriers through thermal jumps that are proportional to kBT with a Fermi statistics property of an acceptance criterion. By contrast, Quantum Annealing addresses the challenge by using quantum fluctuations to tunnel directly through higher, but narrower, energy barriers (Díez-Valle et al., 2026). In practical applications of QUBO problems onto physical annealers (for example, D-Wave Advantage architecture), hardware connectivity constraints often mean that the QUBO graph has to be embedded into exponentially larger physical graph through typically using minor-embedding techniques mapping single logical variables into chains of physical qubits that are connected. Performance is sensitive to correctness in encoding; in fact, the domain-wall encodings can significantly beat the standard one-hot encodings when implemented on sparser physical hardware graphs as they reduce the needed auxiliary physical chains (Qi et al., 2024).

For digital, gate-based architectures, the Quantum Approximate Optimization Algorithm (QAOA) discretizes the adiabatic path into a sequence of parameterized quantum operations (Halffmann & Sharma, 2025). Starting from an equal superposition state $|+\rangle^{\otimes n} = \frac{1}{\sqrt{2^n}}\sum_{\mathbf{x}\in\{0,1\}^n}|\mathbf{x}\rangle$, a p-layer QAOA circuit applies alternating unitaries

driven by the problem Hamiltonian H_C and a mixer Hamiltonian $H_M = \sum_{i=1}^{n} \sigma_i^x$:

$$|\psi(\boldsymbol{\beta},\boldsymbol{\gamma})\rangle = \left(\prod_{l=1}^{p} e^{-i\beta_l H_M} e^{-i\gamma_l H_C} \right) |+\rangle^{\otimes n}$$

where $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_p)$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)$ are continuously tunable variational angle vectors optimized via a classical loop to minimize the expectation value $\mathcal{L}(\boldsymbol{\beta},\boldsymbol{\gamma}) = \langle\psi(\boldsymbol{\beta},\boldsymbol{\gamma})|H_C|\psi(\boldsymbol{\beta},\boldsymbol{\gamma})\rangle$ (Hawashin et al., 2026).

Advanced variants, such as Multi-Angle QAOA (ma-QAOA), assign independent parameter angles to individual terms in the cost and mixer Hamiltonians ($H_C = \sum_{uv} J_{uv} Z_u Z_v \rightarrow e^{-i\gamma_{uv} Z_u Z_v}$), providing fine-grained optimization flexibility at the cost of expanding classical parameter spaces (Keller et al., 2024). Similarly, Parity QAOA maps constraints directly onto continuous hardware configurations, trading two-qubit gate depth against topological covering depths (Pelofske et al., 2023).

3. Trainability Landscapes and Barren Plateau Phenomena

3.1 Mathematical Mechanics of Cost Function Concentration

The foundational barrier to scaling Variational Quantum Algorithms (VQAs) including deep QAOA and Variational Quantum Eigensolvers (VQE) is the Barren Plateau (BP) phenomenon (Gelman, 2024). A barren plateau occurs when the variance of the partial derivatives of the cost function $\mathcal{L}(\boldsymbol{\theta})$ with respect to variational parameters $\boldsymbol{\theta}$ shrinks exponentially as a function of the qubit count n:

$$\mathrm{Var}_{\boldsymbol{\theta}}\left[\frac{\partial \mathcal{L}(\boldsymbol{\theta})}{\partial \theta_\mu}\right] \leq \mathcal{O}(b^{-n}), \quad \text{for some base } b > 1$$

Under this regime, the optimization landscape becomes almost uniformly flat. Because expectation values on physical QPU hardware are evaluated empirically by averaging N_{shots} quantum measurements, resolving non-zero gradient descent directions require $N_{\text{shots}} \in \Omega(b^n)$ quantum executions (Arrasmith et al., 2022). If N_{shots} scales only polynomially with n, shot noise dominates the estimated gradient values, reducing classical parameter updates to an uninformative random walk (Singkanipa & Lidar, 2025).

The flattening of landscapes is caused by three structural design issues in circuit and observable design. In these the mathematical processes behind landscape flattening originate. Here its three. Firstly, Parameterized Quantum Circuits (PQC) high expressive Rapidly become approximate unitary 2-design over the whole 2^n-dimension Hilbert space, forcing expectation values to concentrate around Haar ensemble averages (Schumann et al., 2024). Secondly, any m-local observables, a gauge of fidelity, comparing state across all n qubits, have been identified to introduce exponential concentration even in relatively shallow circuits, on the other hand local observables operating specifically on relevant subsystems keep variance with polynomial bounds (Lan & Huang, 2026). Thirdly, noise processes such as local depolarizing channels that contract intermediate quantum state trajectories to the maximally mixed state create noise-induced barren plateaus that interfere with optimization independent of ansatz geometry (Wang et al., 2024).

3.2 Dynamical Lie Algebras and Expressiveness

A rigorous algebraic framework for analyzing landscape trainability relies on the Dynamical Lie Algebra (DLA) generated by a parameterized quantum circuit (Diaz et al., 2023). Given an ansatz generated by a set of skew-Hermitian operators $\{iH_k\}_{k=1}^{K}$, its DLA $\mathfrak{g}$ is defined as the real Lie algebra spanned by all nested commutators of those generators:

$$\mathfrak{g} = \mathrm{span}_{\mathbb{R}}\left\{iH_k, [iH_j, iH_k], \left[[iH_j, iH_k], iH_l\right], \ldots\right\}$$

The dimension of the DLA, $\dim(\mathfrak{g})$, dictates the operator subspace accessible by the variational ansatz. If $\dim(\mathfrak{g})$ scales exponentially with qubit count n (for instance, $\dim(\mathfrak{g}) \sim \mathfrak{su}(2^n)$), the circuit acts as a 2-design over that manifold, guaranteeing an exponential suppression of gradient variances and triggering an untrainable barren plateau (Adamatti Bridi et al., 2026).

In standard QAOA applied to unweighted Max-Cut problems, the DLA structure depends directly

on graph topology. On path graphs, $\mathfrak{g} \cong \mathfrak{u}(n)$, maintaining polynomial Lie algebraic dimensions (Allcock et al., 2026). However, empirical evaluations across the MQLib benchmark suite demonstrate that for weighted Max-Cut instances with random continuous weight distributions, at least 75% of problem graphs generate a DLA with dimension $\dim(\mathfrak{g}) \geq 2^{128}$, inducing severe landscape concentration deep in the parameter space (Tao et al., 2026).

3.3 Mitigation Strategies and Circuit Ansätze

To prevent the ansatz from covering exponentially large Lie algebras, structural restrictions on the circuit architecture are needed to mitigate barren plateaus Appropriate constraint on circuit depth to logarithmic p∈O(log n) provides a systematic way to achieve this or use Symmetry-awareness to restrict state transformations to smaller Hilbert sub-manifolds (Anselmetti et al., 2021). Non-local parameterization methods (like continuous Fourier-parameterized ansätze) replace discrete step-wise gate variations with smooth, low-frequency dynamic fields and prevent Haar-random state distributions through parameter initialization (Vaquero-Sabater et al., 2025).

Later, Dynamic Parameterized Quantum Circuits (DPQCs) use mid-circuit measurements and dynamic conditional feedback gadgets to dually gate control expressiveness in the quantum model but preserve high target state fidelity (Cunningham & Zhuang, 2025). Although this is done to avert anti-concentration of cost distributions (Holmes et al., 2022), this also involves careful consideration of gadget placement and eventually exercises an un- or undertrained landscape if the feedforward distance between the dynamic operations and target measurement observables is large.

4. Hybrid Quantum-Classical Workflows in Complex Scheduling and Logistics

4.1 Production Scheduling, Vehicle Routing, and Resource Allocation

Industrial scheduling applications such as Job Shop Scheduling (JSP), Capacitated Vehicle Routing Problems (CVRP), and satellite payload acquisition scheduling are constrained by strict temporal and capacity limits. Formulating JSP for QT requires turning the operation start times and machine assignments into a set of binary variables x_(i,m,t) for deciding if operation i would start on machine m at time step t (Liu et al., 2025). \nThe precedence constraints (which require that operation i completes before operation j can start) and resource exclusion constraints (which limit machine m to executing at most a single operation per time step t) are rewritten explicitly as quadratic penalty terms within the QUBO matrix (Shamsuddoha et al., 2025). \nGiven the operational scale of Capacitated Vehicle Routing Problems (CVRP), it uses a two-phase decomposition: a classical clustering phase that groups geographically near delivery nodes, and then a quantum Travelling Salesperson Problem (TSP) solver phase runs on physical annealers or QAOA circuits (Assad et al., 2026). Hybrid solvers are used in automated guided vehicle (AGV) routing models to resolve multi-agent pathing conflicts within seconds for fleets of up to 21 AGVs. In the case of satellite payload acquisition scheduling, a quantum subroutine, embedded in a classical local branch-and-bound engine is used to resolve acquisition requests against orbital pass windows, pointing constraints and power availability during overlapping pass windows (Fedouaki et al., 2026).

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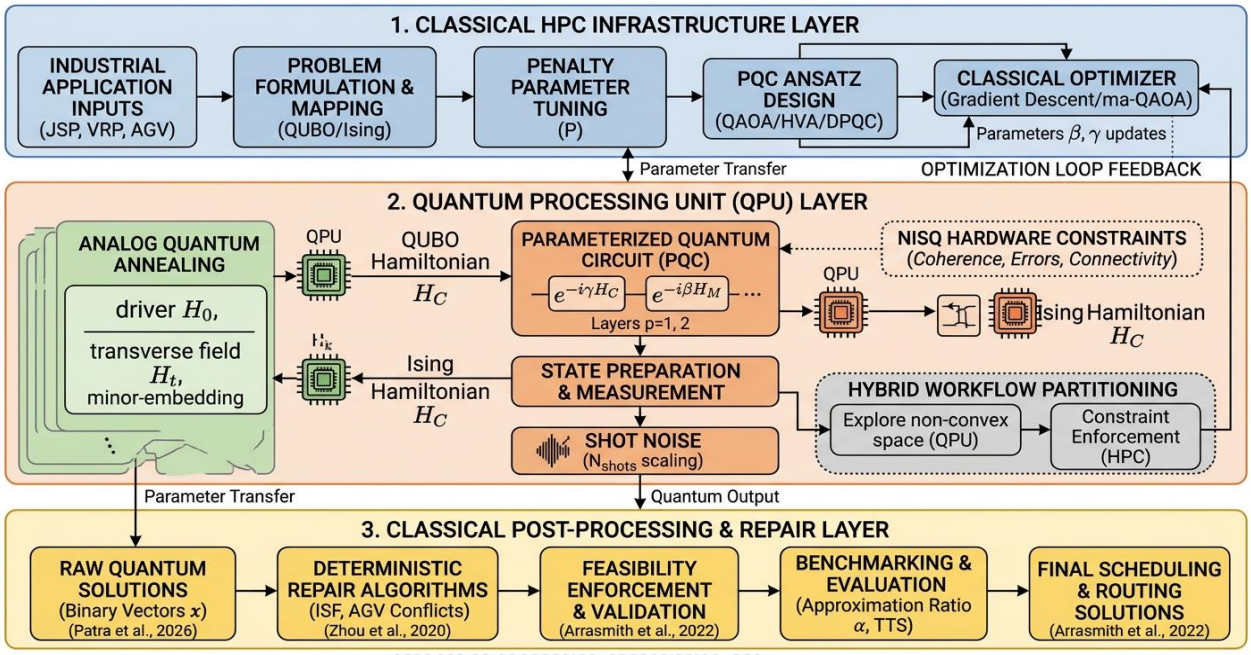


Figure 1: Architecture of a Hybrid Quantum-Classical Optimization Workflow for Industrial Scheduling and Combinatorial Problems.

5. Empirical Benchmarking and Performance Evaluation

To contextualize the current state of hybrid quantum-classical optimization, performance metrics must be benchmarked directly against classical production solvers including Gurobi, Simulated Annealing (SA), Density Matrix Renormalization Group (DMRG) Tensor Networks, and Parallel Tempering (PT) (Allen & McDonald, 2026). Primary evaluation metrics include the **Approximation Ratio** $\alpha = \frac{C(\mathbf{z})}{C(\mathbf{z}_{\text{opt}})}$ (where $C(\mathbf{z}_{\text{opt}})$ is the exact ground-state cut or makespan), **Time-to-Solution (TTS)**, **QUBO Binarization Efficiency**, and **Feasibility Scaling** under dense constraints (Longbottom, 2000).

Table 1: Comparative Benchmarking of Quantum and Classical Optimization Frameworks

Optimization Framework	Solvers / Hardware	Target Class & Benchmark Problem	Approximation Ratio (α\alphaα)	Algorithmic Bottlenecks & Hardware Limitations
Pure Quantum Annealing (QA)	D-Wave Advantage (5000+ qubits), D-Wave 2000Q	Job Shop Scheduling, Unconstrained Max-Cut	$\alpha \approx 0.70 - 0.85$ (on raw QUBOs)	Fails to reach feasible regions on dense constraint systems; minor-embedding chain breaks reduce effectiveness.
Hybrid Quantum-Classical (HQA / QRR*)	Quantum Relax-and-Round (QRR*) + Local Search; Rigetti Ankaa-2	3-Regular Max-Cut ($N = 32$ to 4096)	$\alpha \approx 0.99$ (Average across instances)	Linear algorithmic complexity scaling at layer depth $p = 1$; higher p layers suffer

				from optimization overheads.
Hybrid Multi-Objective QPU Workflow	IonQ Aria-1 Trapped-Ion QPU + ISG / ISF / ISI Heuristics	Airbus Supply Chain (4-KPI Scalarized QUBO)	Traces Pareto-optimal frontier; outperforms classical baselines	Low physical qubit counts require aggressive classical graph reduction (Dijkstra preprocessing).
Gate-Based QAOA (Low Depth)	Direct $p = 1,2$ Circuit Emulators; Transmon QPUs	Signed Max-Cut, Wishart-Planted Graphs ($n = 14 - 48$)	$\alpha \approx 0.69 - 0.75$ (Unassisted $p = 1$)	Parameter optimization landscape encounters narrow gorges; high two-qubit gate error rates degrade output.
Classical Integer Programming	Gurobi Branch-and-Cut Engine	Max-Cut, Dense QUBOs, Industrial MIP Scheduling	$\alpha = 1.00$ (Certified exact on small/medium instances)	Exponential execution time growth (> 600 s) on hard non-convex QUBOs; encounters memory limits.
Classical Physics-Inspired Metaheuristics	Simulated Annealing (SA), Parallel Tempering (PT), DMRG Tensor Networks	MQLib Benchmark Suite, Wishart Instances ($n = 128$)	$\alpha \approx 0.95 - 0.98$	Requires extensive parameter tuning (cooling schedules); tensor networks scale poorly as graph treewidth approaches n.

Empirical solver comparison reveals distinct scaling behaviors across problem sizes. On small instances ($N < 256$), classical exact solvers like Gurobi reliably deliver certified exact solutions ($\alpha = 1.0$). However, as graph size and non-convexity increase to medium scales ($N \approx 4096$), classical exact routines encounter exponential run-time limits, often exceeding 600-second timeouts (Abbas et al., 2024). In this domain, hybrid quantum-classical solvers such as the Quantum Relax-and-Round (QRR*) algorithm augmented with guided local search achieve average approximation ratios of $\alpha \approx 0.99$ while maintaining competitive run-time scaling (Koch et al., 2026).

Map low-energy quantum output into feasible execution schedules for constraint-dense problems, which are inherently intractable to pure analog quantum annealers due to soft penalty definitions that do not correctly capture hard constraints, Bucher et al., 2024). Further, classical parameter optimization is the primary computational cost scaling of variational quantum algorithms. Parameter-transfer strategies provide a way of curtailing this load by first pre-training variational parameters (β, γ) on small, generic Max-Cut instances and then applying them directly to target scalarized instances along a Pareto front of solutions that have higher objective values but maintain low approximation error (Sharma & Lau, 2025).

6. High-Dimensional Data Analysis and Quantum Machine Learning

6.1 Quantum Classification and Anomaly Detection

High-dimensional data analysis is computationally complex for classical machine learning due to the curse of dimensionality. Quantum Machine Learning (QML) sets aside these obstacles by employing parameterized quantum feature map $|\phi(x)\rangle=U(x)|0\rangle^{\wedge}(\otimes n)$ to map classical feature

vectors x∈R^d into a complex Hilbert space H=C^(2^n) (Sai e al., 2026).

Quantum Support Vector Machines (QSVM) embeds feature vectors in high-dimensional Hilbert spaces where previously non-linear boundaries became linearly separable (Oh & Park, 2024). A quantum kernel K (x_i,x_j) =|⟨ϕ (x_i)|ϕ (x_j)⟩|^2, which is defined by the inner products of high-dimensional quantum states, is assessed by quantum state overlap circuits (e.g., the Swap Test). The next step of determining optimal classification hyperplanes is carried out by a classical convex quadratic optimizer that uses the evaluated kernel matrix (Hdaib et al., 2024).

QSVDD creates a sphere of the smallest possible radius around target operational feature representations. In QSVDD, a small set of parameters with a natural logarithmic growth in the number of input qubits O(log n) needs to be learned, which reduces the size of the parameter space needed by classical systems (Belay et al., 2026). In order to tackle trainability challenges, Quantum Convolutional Neural Networks (QCNNs) employ alternating parameterized feature extraction layers and quantum pooling layers that successively coarsen the number of qubits (Rawas, 2026). QCNN architectures apply convolutional filters over neighboring qubit subsystems, reducing their dimensionality; this eliminates the need for Haar 2-design convergence. Consequently, this feature of QCNNs does not exhibit barren plateaus or have to overcome lockstep gradient changes when training (Das et al., 2024).

6.2 Quantum Principal Component Analysis and the Dequantization Framework

Quantum Principal Component Analysis (qPCA) has historically been regarded as a prime demonstration of exponential quantum speedup for high-dimensional data analysis. Given an unknown density matrix ρ ∈ ℂ^(N×N) that corresponds to a data covariance matrix, qPCA operates on the matrix exponential e^(−iρt) (Patra et al., 2026) using phase estimation techniques. This analysis retrieves the largest eigenvalues λ_k and projects input states onto the corresponding principal eigenvectors |v_k⟩ in time O ((log N) ²), compared with the classical Singular Value Decomposition (SVD) runtime of $O(N^3)$ (Fazlioglu, 2026). This theoretical exponential speedup, however, relies on very strong data-loading assumptions, most notably the requirement that classical data be pre-stored in a Quantum Random-Access Memory (QRAM) data structure that permits quantum state preparation of |ψ⟩ = Σ_(i=1)^N x_i |i⟩ in O(log N) time (Thakur et al., 2025).

Ewin Tang devised the classical dequantization framework grounded in sample-and-query (SQ) access to assess such claims. In the classical model of SQ access, a query can be an O (1)-time operation to query any A_ij, to sample rows (i) with probability proportional to their ℓ_2-norm $\|A_{(i,*)}\|^2/\|A\|_F^2$, and to sample entry indices within a row with probability proportional to their squared magnitude $|A_{ij}|^2/\|A_{(i,*)}\|^2$ (Vani, 2026). Tang used SQ access to create classical randomized algorithms for qPCA, recommendation systems, and k-means clustering. These classical algorithms efficiently reconstruct low-rank matrix approximations through sampling of small sub-matrices whose dimensions depend only on the matrix rank, k, and the error tolerance, ε, rather than on the ambient dimension, N (Acampora et al., 2025).

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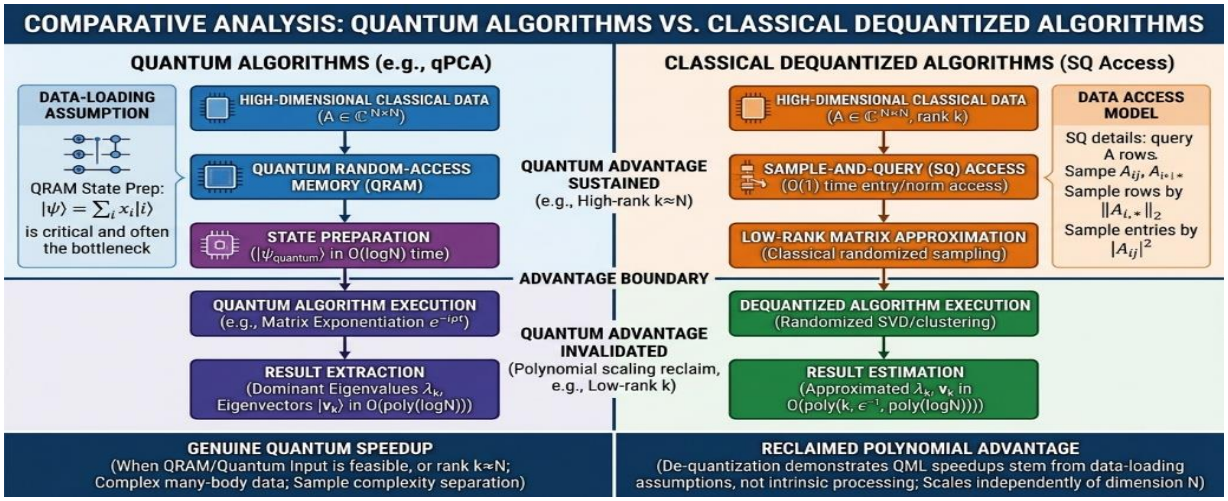


Figure 2: Comparative Data Pipeline and Complexity Boundaries Between Quantum Machine Learning (QML) and Classical Dequantization Frameworks.

Table 2: Comparative Framework for High-Dimensional QML Algorithms and Classical Dequantization

QML Paradigm	Quantum Time Complexity	Data-Loading Access Model	Classical Dequantized Time Complexity	Reclaimed Advantage Status
Quantum Recommendation Systems	$\mathcal{O}(\text{poly}(k)\log(MN))$	QRAM State Prep	$\mathcal{O}\big(\text{poly}(k)\text{polylog}(MN)\big)$	**Polynomial Only** (Exponential claim invalidated by Tang)
Quantum Principal Component Analysis (qPCA)	$\mathcal{O}((\log N)^2)$	QRAM Density Matrix State	$\mathcal{O}\big(\text{poly}(k,\epsilon^{-1})\text{polylog}(N)\big)$	**Polynomial Only** (Demonstrated polynomial bound for low-rank k)
Quantum Supervised Clustering / Centroid	$\mathcal{O}(\text{poly}(k)\log N)$	SQ Query / State Prep	$\mathcal{O}\big(\text{poly}(k,\epsilon^{-1})\text{polylog}(N)\big)$	**Polynomial Only** (Dequantized via low-rank sampling)
Quantum Support Vector Machine (QSVM)	$\mathcal{O}(\text{poly}(S)\log N)$	Quantum Feature Maps $U(\mathbf{x})$	$\mathcal{O}\big(\text{poly}(S,N)\big)$ (Dependence on classical kernel construction)	**Problem Dependent** (Advantage depends on classical hardness of target kernel)

Quantum Anomaly Detection (QSVDD / QCNN)	$\mathcal{O}(\text{polylog}(N))$	Quantum Circuit Convolution	Not Dequantized (Sustained via localized entanglement & low DLA)	**Sustained NISQ Utility** (Avoids barren plateaus via local m-qubit pooling)

6.3 Boundaries of Genuine Quantum Advantage

Dequantization analysis shows that speedups in QML are typically polynomial as opposed to exponential when equivalent data access models are chosen for classical low-rank matrices. This study of an out-of-reach area for classical algorithms delineates where quantum advantage belongs (Nassir et al., 2025). The dequantized classical algorithms are a time complexity O ("poly" (k) that is, proportion to the rank of the matrix. Unlike their classical counterparts that falter on high-rank or full-rank matrices (k≈N), quantum sampling routines hold and restore the quantum advantage (Madhavi et al., 2026). Additionally, when physical quantum systems (such as quantum sensors, atomic clocks, or complex many-body physical reactions) serve as the source of input data, the state $|\psi,"quantum"\rangle$ is intrinsically present in the QPU registers (Keçeci, 2025). When taken in context with the fact that Exponential number of physical measurements O(2^n) state tomography must be done for classical algorithms to process this data, QML architectures that accept direct quantum transductions offer an exponential sample complexity separation for learning physical properties of nature (Raseena, 2025).

Conclusion

This review has looked at the theory the way the algorithms are built and how well hybrid quantum and classical optimization and Quantum Machine Learning work in scheduling, complex optimization and big data analysis. The study leads to a main point. First the QUBO and Ising Hamiltonian methods offer a way to connect classical problems with quantum machines. Adjusting the penalty numbers and changing higher order problems to quadratic forms are still big problems. If the penalty numbers are too low the answers don't work. If they are too high the problems become too hard for both quantum and classical machines making them stuck in solutions. Changing order problems to quadratic forms makes too many variables and uses up the limited qubits. Second the quantum and classical methods have real benefits now. On medium-size Max-Cut problems the Quantum Relax-and-round methods get very close to the answers with good speed. Pure quantum machines often fail on complicated problems unless they use help from classical methods. Optimizing the numbers in methods is still the main problem in the hybrid methods, which makes it important to share these numbers across different solutions. Third the Barren Plateau problem is the issue for the learning ability of the quantum methods. Looking at the way the math works shows that the way the problem is set up controls how the learning works. Least 75% of the weighted Max-Cut problems create very big math problems. Using circuits, better ways to build the quantum models and looking at small parts of the data help keep the learning stable. The QCNN structures avoid the Barren Plateau problem by using parts and grouping them. Fourth the idea of making quantum methods work without quantum parts changes the way we think about quantum advantage. For data the quantum methods, like qPCA, quantum recommendation systems and quantum clustering do not give a big speed boost. The real advantage only happens when the data is very complex or comes from quantum sources. Future research should focus on making the circuits better at handling noise using ways to test and compare combining better classical help with the quantum methods and understanding exactly where the advantage comes from. Hybrid quantum and classical computing is a way to get useful results now in scheduling and medium size problems while we continue to find

where real quantum advantage exists through more research and better machines.

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