

EFFECT OF PRESSURE RATIO ON COP OF THE VORTEX TUBE REFRIGERATION ON DIFFERENT MASS FRACTIONS OF THE AIR STREAMS

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Abstract

Environment degradation is enhancing by using the chlorofluorocarbons (CFCs) and hydrochlorofluorocarbons (HFCs) as refrigerants in the heating, ventilation and air conditioning (HVAC) systems. Vortex Tube (VT) refrigeration is one of the cheapest techniques used for producing refrigeration effect. VT is a simple device (does not have any moving part) that generates the cold and hot streams of air/gas simultaneously when the pressurized air/gas is injected through it. Recently, studies have been carried out to improve the Co-efficient of Performance (COP) of the VT Refrigerator. There are many parameters, as literature shows, that effect the COP of VT. Pressure Ratio (Ratio of the pressure at which air/gas is injected to the VT to the outside Pressure where the compressor is placed), which is an important parameter of thermodynamics and plays an essential role when energies change from pressure to velocity, has not been discussed yet in the literature. This paper discusses the effect of pressure ratio on different air streams affecting the COP of VT Refrigeration. In conclusion, this research shows, as the pressure ratio increases the COP decreases on the different mass fractions of air/gas and the Regional effect of pressure ratio is also shown in the Result and Discussion section.

INTRODUCTION

The vortex tube or sometimes referred to as Ranque Hilsch vortex tube is a simple mechanical device that does not have any moving or electrical parts and can generate up to 6000 BTUs of refrigerating effect¹. The VT refrigeration uses compressed air to produce hot and cold airstreams at either end of the apparatus². This is achieved by injecting air stream tangentially into the tube at high speed with the help of nozzles. Highly pressurized gas is tangentially injected into a swirl chamber and accelerated to a high rate of rotation. At the end of the tube, only the outer shell of the

compressed gas can escape due to the conical nozzle. The remainder of the gas returns in an inner vortex of reduced diameter within the outer vortex. The separation of the air stream into two parts is another matter³. In the tube, separation of hot and cold streams takes place and both streams are collected at either end of the tube; hence named, hot end and the cold end of a VT⁴. The schematic of an actual VT has been shown in figure 1. Because of its simple geometry and ease of manufacturing, it is quite popular in the scientists for a while now⁵.

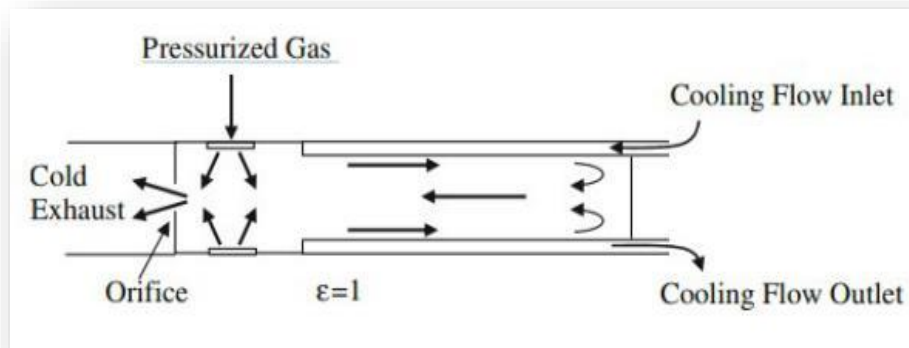


Figure 1. Schematic of VT

Many research papers have been written on the working, experimental modulation, thermal separations, thermal acoustics and application of the VT⁶⁷⁸.

G.J Ranque invented the VT and was studied in detail by Hilsch. Hence, sometimes it is also referred to as Ranque-Hilsch VT. The initial reaction to this discovery was not so good. The VT invented at that time was highly inefficient. This device remained abandoned for quite a period until Hilsch studied it in detail in 1947. He carried out his own detailed experiments and aimed at improving the thermodynamic efficiency of the VT. Due to the efforts of Hilsch and other scientists who followed him, VT now finds applications not only in the small places but also in industrial applications too. The VT consists of a principal tube with three openings. The opening that lies on the circumference allows the compressed air from the compressor to enter the vortex tube. The other two lie on the circular cross-section of the tube. The cross-sectional opening that is nearer to the circumferential opening serves as the cold exit and the other one serves as the hot air exit. The hot air exit opening has a conical valve that controls the flow of hot air coming out of the opening. Once the compressed air falls tangentially on the inner walls of the tube, it expands adiabatically. The sudden expansion results in the formation of the cool air near the axis of the vortex tube. Now after entering the VT,

the air splits into two regions i.e. thermal separation takes place⁹ as shown in Fig 1. One that is nearer to the axis is cooled air and the other that is nearer to the walls of the tube is heated air. There are several theories and researches explaining the thermal separation of air and an increase in efficiency based on parameters like nozzle angle inside the VT¹⁰¹¹. Many research papers have also been written on them. One of the theories explains the thermal separation using the frictional effects of the walls and the fluid.

The layers of the fluids that are nearer to the walls of the vortex tube are heated as the air is swirling at around 2 million RPM inside the vortex tube and the friction between the two surfaces raises the temperature but as we move towards the axis of the tube the temperature drops. The flow can be similar to that of a pipe. The hot air exits the opening having the conical valve and cold air is forced to leave the other end¹².

1.1. Choking

In Dynamics, the choked of fluid is a condition associated with the venturi effect. When fluid flows from converging objects at high pressure to low pressure, the fluid velocity increases¹³. At initially subsonic upstream conditions, according to the law of conservation of mass, the fluid velocity requires to increase. At the same time, the venturi effect causes the static pressure and density to decrease. The physical point at which choking

occurs for adiabatic conditions is when the exit plane velocity is at sonic i.e. Mach number is 1¹⁴. Moving conical valve forward and backward, as shown in Figure 2, can help to control the flow of hot and cold air. There is a point up to which the valve can be moved forward because at that point the backpressure at the exit of the hot air and the inlet pressure becomes equal and the condition of choking is achieved¹⁵. At this choking condition, the hot air's flow rate cannot be further restricted. If the valve is moved more forward, then the hot air starts mixing with the cold air and the refrigeration effects start to drop¹⁶.

1.2. Problem Statement and Objective

One main thing that needs to be discussed here is the entry process of the air into the vortex tube. The process followed by the air at the entry is a highly irreversible process and is adiabatic in nature. The sudden expansion can be best explained as the throttling condition. The amount of work done on the system is given by the equation¹⁷;

$$\dot{W}_{in} = \dot{m}(h_{inlet} - h_{outlet}) \dots (1)$$

Where $\dot{W}_{in}$ is the work done per second on the gas by the Compressor and h_{inlet} , h_{outlet} are the specific enthalpies at the inlet and exit of the air/gas. But here the entry process is highly irreversible and the enthalpy at that point cannot be calculated. So, the formulae that are used in this case is¹⁸;

$$\dot{Q} = \dot{m}c_p\Delta T \dots (2)$$

Here the ΔT is difference in temperature, can easily be found using temperature sensors. And c_p is the specific heat capacity of gas which is air in this case. Equations 1 and 2 are used to calculate the input work done/second on the air by the compressor used for pressurized air since these equations are only valid for the Closed systems (Control Mass system). These equations cannot be used to calculate the input work done/second at any point inside of the VT because VT is an open system (Control Volume System). Previously research papers have been written on studying the effect of inlet pressure change on the refrigerating effect and thermodynamic changes in the refrigerant¹⁹. In 1988 R.T. Balmer applied liquid water as the working medium. It was found that

when the inlet pressure is high, for instance, 20-50 bars, the heat energy separation process exists in incompressible (liquids) vortex flow as well. Note that this separation is only due to heating; there is no longer cooling observed since cooling requires compressibility of the working fluid. When the inlet pressure increases, the temperature difference in the cold end and hot end is increased. When the inlet pressure increases, the Carnot COP is decreased²⁰. Hence, VT can be used for any type of spot cooling or spot heating application. But this paper allows us to see what changes occur when atmospheric pressure involvements are brought in the working of the VT. The current paper discusses the effects of pressure ratio on the Refrigeration Effect (RE) and COP of the VT. Experimental modulation has been done and the optimum pressure ratio is found. The initial part of the experiment is conducted in the RAC lab of the Mechanical Engineering Department, University of Engineering and Technology, Lahore Pakistan.

And the later part is conducted at altitude, Murree Pakistan. The demographic change in location allows us to see the results of the experiment in detail. Pressure ratio increases in Murree and so does compressor does work for the same pressures at which the air is injected into the VT.

Experimentation

The original experiment was to check the optimum pressure ratio where the VT would best work. To calculate that a range of cold mass fractions was to be calculated that will give us the maximum COP under normal conditions. After finding the optimum cold fractions through experiments as shown in Table 1, a set of experiments was conducted on three different inlet pressures to see where the maximum COP is achieved. Refer to Table 1, the experiments were performed on 300 kPa, 400 kPa, and 500 kPa. Unfortunately, the experiment for the latter two pressures could not be completed due to some equipment limitations.

But the experiment on the inlet pressure of 300 kPa reveals a cold mass fraction range of 0.5-0.6 is optimum to run the VT. This trend is also seen in the 400 kPa and 500 kPa inlet pressures. The COP

of the system is increasing as we increase the cold mass fraction, but we had to assume that COP would be maximum too in the range where cold mass fraction lies between 0.5-0.6. Our latter experiments are conducted on this cold mass fraction range and the comparison between pressure ratios and COP is analyzed. The optimum cold mass fraction at the 300 kPa

pressure is 0.57. this value is precisely accurate reviewing all the literature that has been done on the vortex tube. The literature review validates this experiment and the effect of pressure ratios on COP can be studied on these cold mass fraction ranges as shown in the forms of Tables and Graphical Figures.

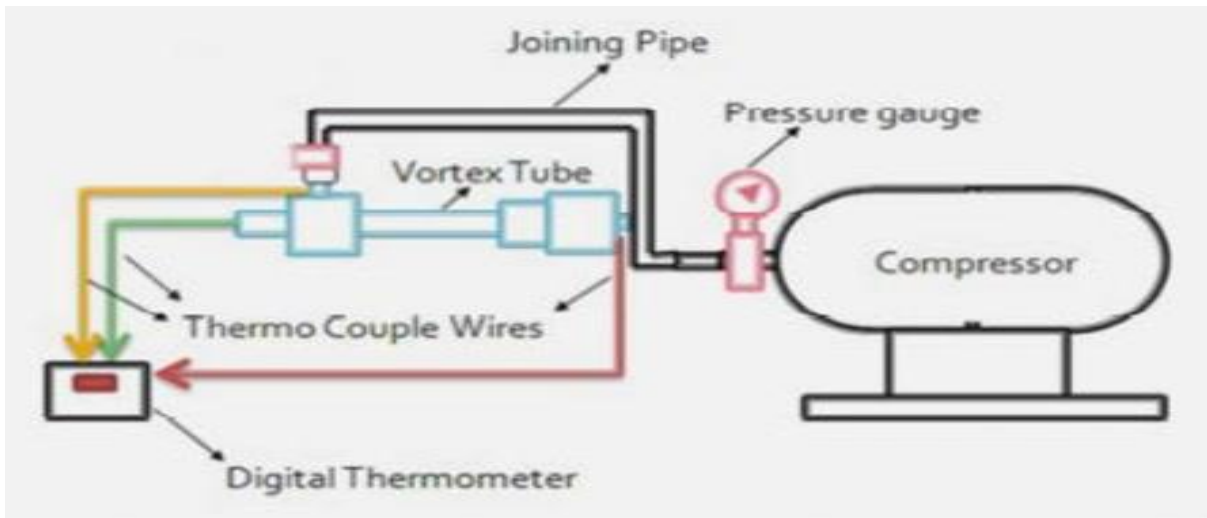


Figure 2. Experimental setup of VT

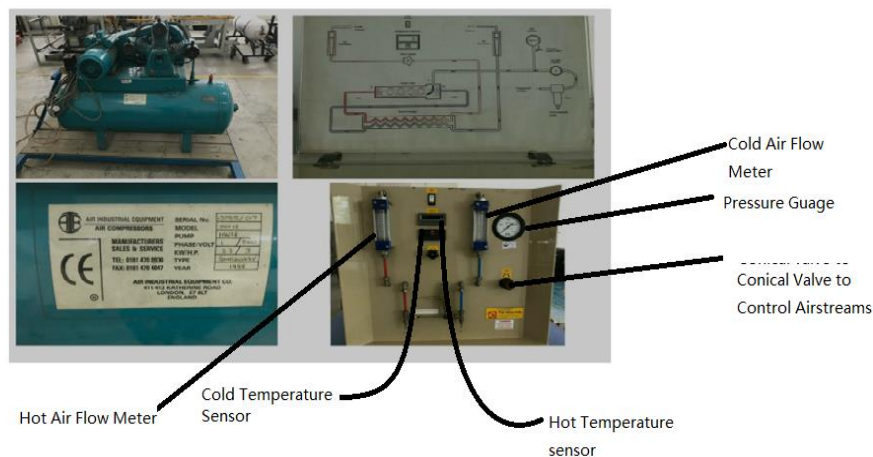


Figure 3. Labeled Vortex Tube along with the Compressor

Figure 3. represents the Parallel VT Refrigeration Unit long with the compressor which has been used in the experimentation. For**Values in Tables 1,2,3,4,5, Readings of the temperatures were taken by using the temperature sensors available on the apparatus one for Hot Stream and the other one for Cold Stream. Airflow meter with red inlet provides the values of hot streams air i.e. Hot mass

**The Values in the Tables 1,2,3,4,5 have been calculated in Microsoft Excel by considering the Numbers of Significant Digits (SD) to avoid the Round Off errors in the calculations. Basic formula to calculate

fractions while the cold fractions are measured by the air flowmeter with blue inlet and the quantity of the cold/hot mass fractions is controlled by the black color conical valve at the right bottom. At the right top, the pressure at which the air (working fluid in our discussion) is injected into the VT is measured by the Bourdon Pressure Gauge.

the SD was used, which is as follow : $|X - X^*| \geq .5 \times 10^{m-n}$ where X =value calculated by Excel (Approximated actual value), X^* = value calculated manually, m is power exponent of 10 in the value calculated manually, and n represents Numbers of SDs.

2.1. Experimental Readings Part(A)

This part of the experiment was conducted in Lahore, Pakistan using the VT Refrigeration

System shown in Figure 3, placed in RAC Lab UET Lahore.

Table 1. Highlighted shells show the optimum cold mass fraction for their respective inlet pressures

Sr. No	Inlet Pressure	Inlet Temp.	Cold Air Temp.	Hot Air Temp	Cold Air Flow Rate	Hot air Flow Rate	Cold Air Fraction	Refrigeration Effect	Work Input	Coefficient of Performance
	P_1	t_1	t_2	t_3	$\dot{m}_2$	$\dot{m}_3$	$\frac{\dot{m}_2}{\dot{m}_2 + \dot{m}_3}$	RE	$\dot{W}_{in}$	$COP_{RE} = \frac{RE}{\dot{W}_{in}}$
	Kpa	$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	Kg/s	Kg/s		Watt	watt	
1	300	22.6	-1.8	28.2	1	1.8	0.357143	24.522	301.4202	0.08135487
2	300	22.3	-3.1	29.5	1.2	1.7	0.413793	30.6324	311.8684	0.0982222
3	300	22.7	-3.5	30.4	1.4	1.6	0.466667	36.8634	323.0594	0.11410719
4	300	23	-4.6	32.8	1.6	1.6	0.5	44.3808	344.9463	0.12866003
5	300	22.9	-5.5	35.3	2	1.5	0.571429	57.084	377.1576	0.15135317
6	300	22.9	-6	36.9	2.1	1.5	0.583333	60.99345	387.9335	0.15722656

7	400	23.8	-1	25.6	.4	2.6	0.1333 33	9.9696	428 .38 95	0.023272 28
8	400	23.6	-1.8	26.8	.7	2.5	0.2187 5	17.8689	456 .64 08	0.039131 19
9	400	23.9	-2.3	28.4	1	2.3	0.3030 3	26.331	471 .38 72	0.055858 54
10	400	23.9	-4.5	29.4	1.2	2.3	0.3428 57	34.2504	499 .95 61	0.068506 81
11	400	23.9	-5.5	30.7	1.4	2.2	0.3888 89	41.3658	514 .24 05	0.080440 57
12	500	25	-5.2	28.7	1.1	2.8	0.2820 51	33.3861	672 .52 86	0.049642 65
13	500	25.1	-4.9	29.4	1.2	2.8	0.3	36.18	690 .00 44	0.052434 45
14	500	25.5	-5.6	30.6	1.5	2.7	0.3571 43	46.8825	725 .47 68	0.064623 02
15	500	25.7	-6.8	31.5	1.7	2.7	0.3863 64	55.52625	760 .53 25	0.073009 7
16	500	26.6	-9	33.5	2	2.6	0.4347 83	71.556	797 .49 79	0.089725 63

The experiments in Table 1. were performed to obtain the optimum cold fractions. Now we will Check the effect of Pressure Ratio by changing the mass fractions in the following tables along with the graphical figures for Lahore, Pakistan.

Table 2. Effect of Pressure ratio on COP at inlet pressure of 300Kpa

Sr. No	Suction Pressure	Inlet Pressure	Pressure Ratio	Inlet Temp.	Cold Exit Temp.	Hot Air Temp.	Cold Air Flow rate	Hot Air Flow rate	Cold Air Fraction	Refrigeration Effect	Work Input	Coefficient of Performance
	P	P_1	$\frac{P}{P_1}$	t_1	t_2	t_3	$\dot{m}_2$	$\dot{m}_3$	$\frac{\dot{m}_2}{\dot{m}_2 + \dot{m}_3}$	RE	$\dot{W}_{in}$	$COP_{RE} = \frac{COP_{RE}}{\dot{W}_{in}}$

	Kpa	Kpa		°C	°C	°C	Kg/s	Kg/s		watt	Watt	
1	101.3 25	300	2.960 76	22.6	-1.8	28.2	1 8	1. 8	0.357 142	24.5222	301.42 02	0.0813 56
2	101.3 25	300	2.960 76	22.3	-3.1	29.5	1. 2	1. 7	0.413 793	30.6324	311.86 84	0.0982 22
3	101.3 25	300	2.960 76	22.7	-3.5	30.4	1. 4	1. 6	0.466 666	36.8634	323.05 94	0.1141 07
4	101.3 25	300	2.960 76	23	-4.6	32.8	1. 6	1. 6	0.500 000	44.3808	344.94 63	0.1286 6
5	101.3 25	300	2.960 76	22.9	-5.5	35.3	2	1. 5	0.571 428	57.0840	377.15 76	0.1513 53
6	101.3 25	300	2.960 76	22.9	-6	36.9	2. 1	1. 5	0.583 333	60.9934	387.93 35	0.1572 26

Table 3. Effect of Pressure ratio on COP at inlet pressure of 400Kpa

Sr. No	Suction Pressure	Inlet Pressure	Pressure Ratio	Inlet Temp.	Cold Exit Temp.	Hot Air Temp.	Cold Air Flow rate	Hot Air Flow rate	Cold Air Fraction	Refrigeration Effect	Work Input	Coefficient of Performance
	P	P_1	P_1/F	t_1	t_2	t_3	$\frac{\dot{m}_2}{\dot{m}_2 + \dot{m}_3}$	$\dot{m}_3$	RE	W_{in}	$COP_{RE} = \frac{RE}{W_{in}}$	
	Kpa	Kpa		°C	°C	°C	Kg/s	Kg/s		watt	Watt	
1	101.325	400	3.94769	21.9	1.7	23.7	0.4	2.5	0.13793	8.1204	411.459	0.01974
2	101.325	400	3.94769	21.5	1.2	24.0	0.5	2.4	0.17241	10.2001	410.901	0.02483
3	101.325	400	3.94769	21.4	-0.3	24.5	0.7	2.3	0.23333	15.2659	424.925	0.03593

4	101.3 25	400	3.947 69	21.5	-0.7	24.9	0. 8	2. 2	0.266 667	17.8488	425.07 0	0.0419 9
5	101.3 25	400	3.947 69	21.6	-1.8	25.5	0. 9	2. 2	0.290 322	21.1653	439.38 8	0.0481 7
6	101.3 25	400	3.947 69	21.8	-2.1	26.2	1. 0	2. 15	0.317 460	24.0195	446.77 8	0.0537 6
7	101.3 25	400	3.947 69	22.2	-2.5	27.2	1. 1	2. 1	0.343 750	27.3056	454.48 5	0.0600 8
8	101.3 25	400	3.947 69	22.3	-2.9	27.5	1. 2	2. 1	0.363 636	30.3912	468.84 7	0.0648 2
9	101.3 25	400	3.947 69	22.2	-3.1	28.0	1. 3	2. 0	0.393 939	33.0544	468.68 8	0.0705 3
10	101.3 25	400	3.947 69	22.2	-3.5	29.5	1. 4	2. 0	0.411 764	36.1599	482.89 1	0.0748 8

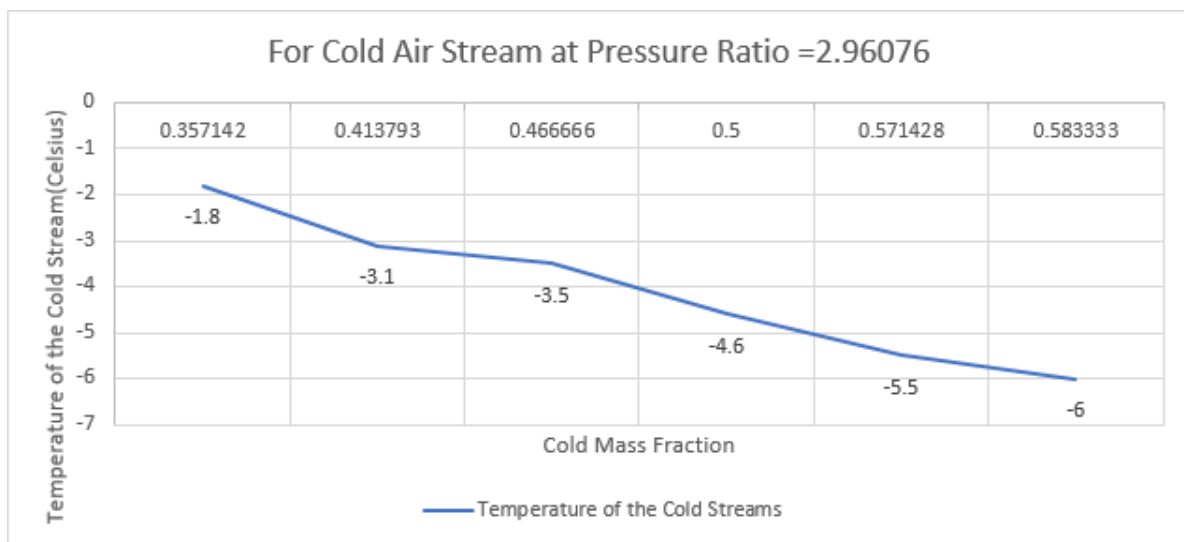


Figure 4.Cold Mass Fraction Vs Temperature of the Cold Stream (Pressure Ratio=2.96)

This Figure shows the behavior of the air. The graph is in accordance with the literature studied for the vortex tube. The temperature of the cold

air goes on decreasing as we increase the cold mass fraction up till a cold mass ratio of 0.57 was obtained.

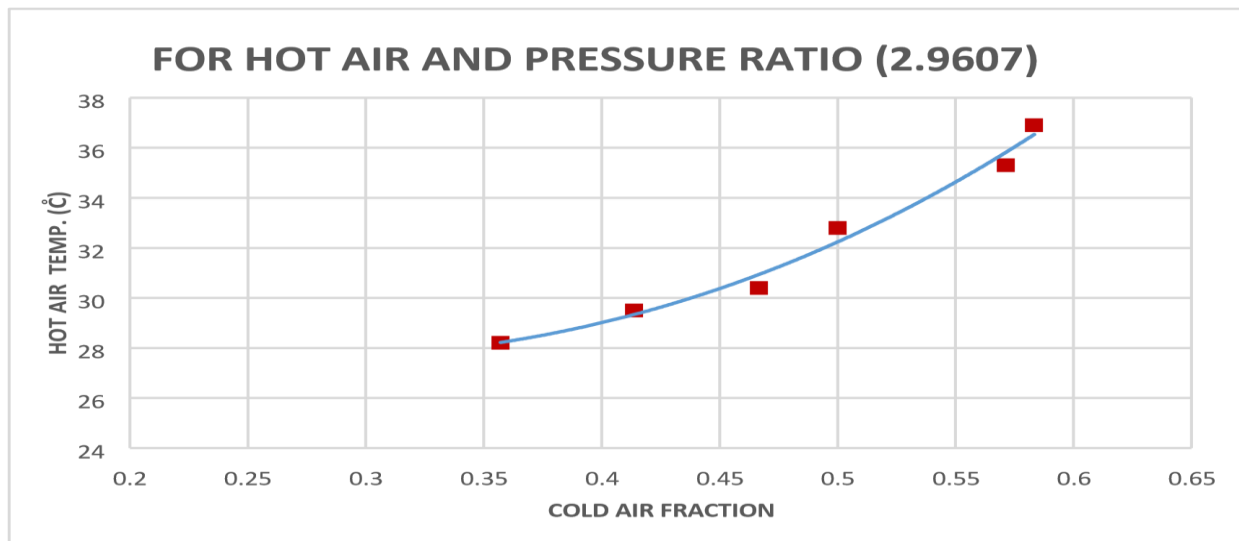


Figure 5. Cold Mass Fraction Vs Hot Temperature (Pressure Ratio=2.96)

This figure represents the trend of hot air at the same ratio of cold mass fraction it also shows data depiction is in accordance with the literature. The

temperature of the hot air goes on increasing as we increase the cold mass fraction up to a ratio of 0.57.

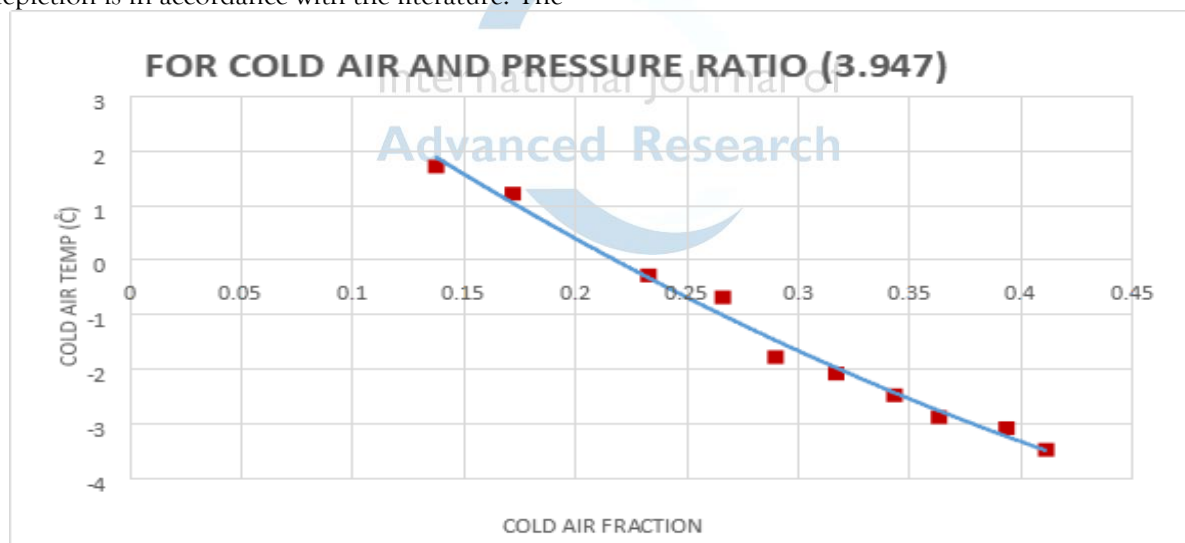


Figure 6. Cold Air Temperature Vs Cold Air Fraction (Pressure Ratio=3.94)

The trend of cold air at 400 kPa, a pressure of ratio of 3.947, is also in accordance with the literature and research studied on the vortex tube.

The temperature of the cold air goes on decreasing. It would have increased after the cold mass fraction of 0.58 had been achieved.

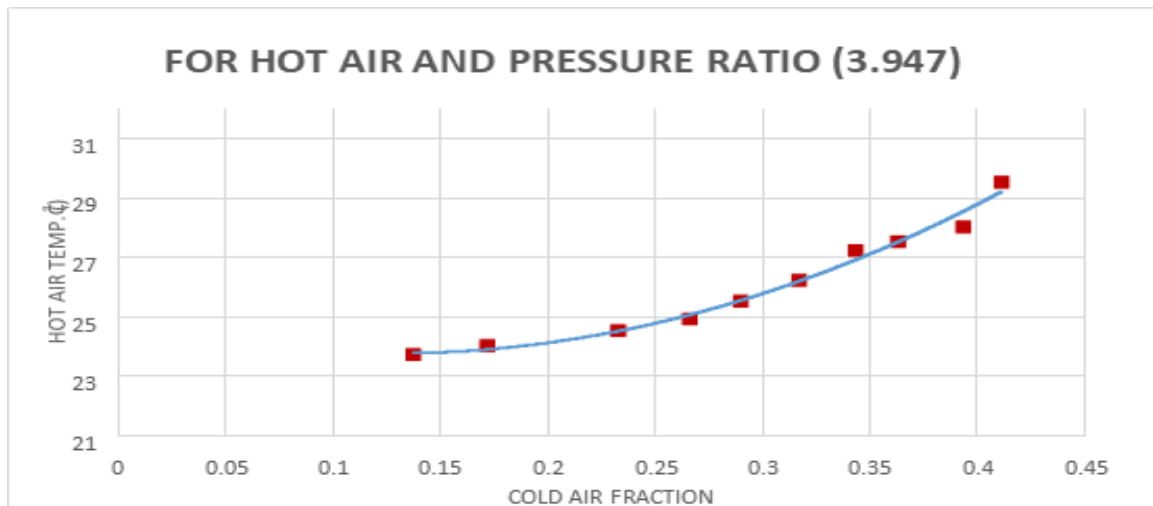


Figure 7. Hot Air Temperature Vs Cold Air Fraction (Pressure Ratio=3.94)

The trend of hot air at a pressure ratio of 3.947 shows that the temperature of hot air goes on increasing. This figure also depicts the same results

as depicted by the previous researches carried on vortex tubes.

2.2. Experimental Readings Part(B)

This part of the experimentation was conducted in Murree, Pakistan using the same apparatus at the same inlet pressures that were there in Lahore i.e. 300kpa and 400kpa. The only difference was

found to be in COP and in Work input, $\dot{W}_{in}$. Work input is increased, and the COP is decreased

Table 4. Effect of Pressure ratio on COP at inlet pressure of 300Kpa

Sr. No	Suction Pressure	Inlet Pressure	Pressure Ratio	Inlet Temp.	Cold Exit Temp.	Hot Air Temp.	Cold Air Flow rate	Hot Air Flow rate	Cold Air Fraction	Refrigeration Effect	Work Input	Coefficient of Performance
	P	P_1	$\frac{P_1}{P}$	t_1	t_2	t_3	$\dot{m}_2$	$\dot{m}_3$	$\frac{\dot{m}_2}{\dot{m}_2 + \dot{m}_3}$	RE	$\dot{W}_{in}$	$COP_{RE} = \frac{RE}{\dot{W}_{in}}$
	Kpa	Kpa		°C	°C	°C	Kg/s	Kg/s		watt	Watt	
1	86.659	300	3.461844	22.6	-1.8	28.2	1	1.8	0.35714	24.522	353.041	0.0694
2	86.659	300	3.461844	22.3	-3.1	29.5	1.2	1.7	0.4137	30.6324	365.278	0.0838
3	86.659	300	3.461844	22.7	-3.5	30.4	1.4	1.6	0.4666	36.8634	378.386	0.0974

4	86.659	300	3.461844	23	-4.6	32.8	1.6	1.6	0.5000	44.3808	404.021	0.1098
5	86.659	300	3.461844	22.9	-5.5	35.3	2.0	1.5	0.5714	57.084	441.749	0.1292
6	86.659	300	3.461844	22.9	-6.0	36.8	2.1	1.5	0.5833	60.9934	454.37	0.1342

Table 5. Effect of Pressure ratio on COP at inlet pressure of 400Kpa

Sr. No	Suction Pressure	Inlet Pressure	Pressure Ratio	Inlet Temp.	Cold Exit Temp.	Hot Air Temp.	Cold Air Flow rate	Hot Air Flow rate	Cold Air Fraction	Refrigeration Effect	Work Input	Coefficient of Performance
	P	P_1	P_1/P	t_1	t_2	t_3	$\dot{m}_2$	$\dot{m}_3$	$\frac{\dot{m}_2}{\dot{m}_2 + \dot{m}_3}$	RE	$\dot{W}_{in}$	$COP_{RE} = \frac{RE}{\dot{W}_{in}}$
	Kpa	Kpa		$^{\circ}C$	$^{\circ}C$	$^{\circ}C$	Kg/s	Kg/s		watt	Watt	
1	86.659	400	4.615792	21.9	1.7	23.7	0.4	2.5	0.13793	8.1024	469.35	0.01730
2	86.659	400	4.615792	21.4	1.2	24	0.5	2.4	0.17243	10.201	468.32	0.0217
3	86.659	400	4.615792	21.4	-0.3	24.5	0.7	2.3	0.23333	15.265	484.45	0.0314
4	86.659	400	4.615792	21.5	-0.7	24.9	0.8	2.2	0.26667	17.848	484.32	0.0368
5	86.659	400	4.615792	21.6	-1.8	25.5	0.9	2.2	0.29032	21.165	501.64	0.0426
6	86.659	400	4.615792	21.8	-2.1	26.2	1.0	2.15	0.31746	24.019	509.43	0.0471
7	86.659	400	4.615792	22.2	-2.5	27.2	1.1	2.1	0.34375	27.312	518.34	0.0526
8	86.659	400	4.615792	22.3	-2.9	27.5	1.2	2.1	0.36363	30.391	534.81	0.0568
9	86.659	400	4.615792	22.2	-3.1	28	1.3	2.0	0.39393	33.054	534.63	0.0618
10	86.659	400	4.615792	22.2	-3.5	29.5	1.4	2.0	0.41176	36.159	550.15	0.0656

Results and Discussions

The Comparison of COPs, at different Pressure Ratio by changing the Mass Fractions on two Different Areas, is in the following Figures.

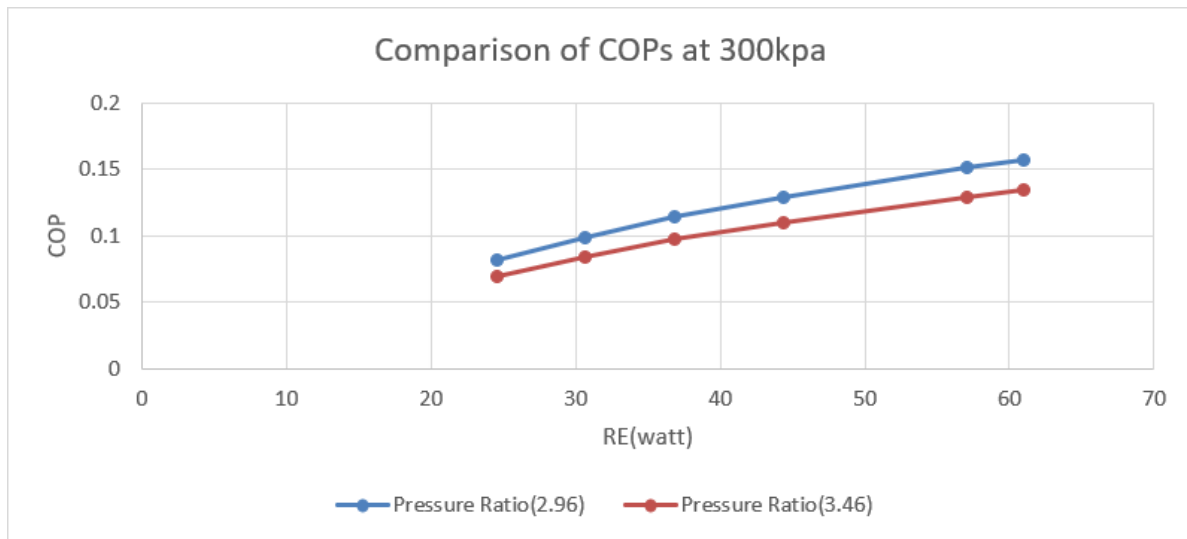


Figure 8. COP Vs RE at 300kpa

This shows the comparison of COPs at 300kpa in Lahore and Murree and, at 300kpa inlet pressure to the VT, the pressure ratio in the Lahore is 2.96

while that in the Murree is 3.46 i.e. pressure ratio is increased in the Murree for the same inlet pressure of the VT, hence the COP is decreased.

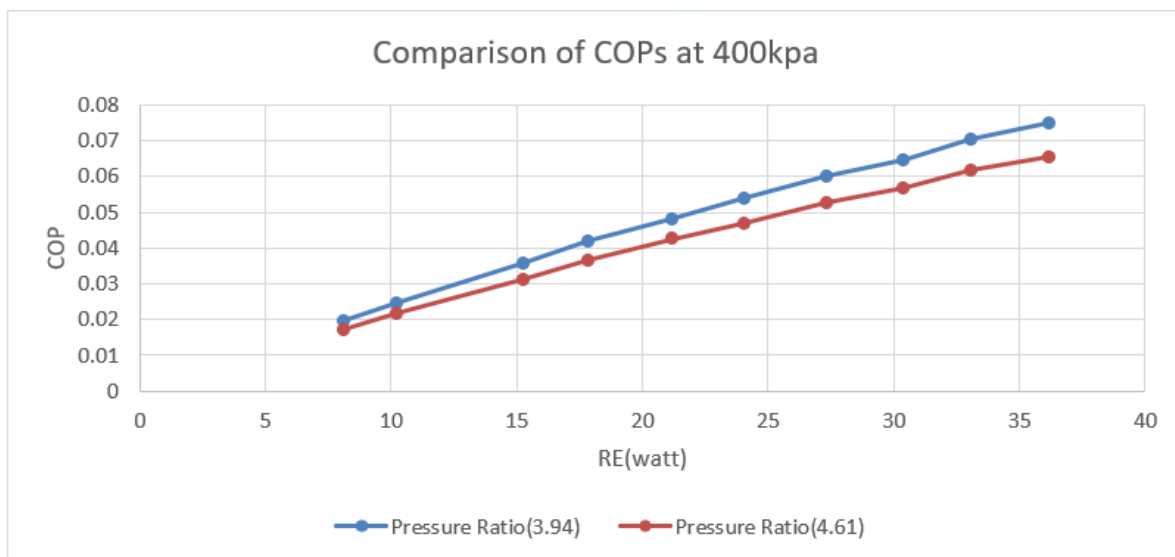


Figure 9. COP Vs RE at 400kpa

This shows the comparison of COPs at 400kpa and the trend is the same as seen for the 300kpa in Figure 8. As the inlet pressure increases from 300kpa to 400kpa the difference in the COPs in these two regions becomes smaller.

Conclusions

(1) When the graph between COP and Work Input is compared before and after changing the pressure ratios i.e. changing the external pressure we see that the COP is decreased, and the work

(2) If work input is increased, then COP would decrease per their formulas. This is not only verified theoretically but also experimentally. Figures and 9 shows the comparison between the two pressure ratios at 300 kPa and 400 kPa.

(4) In the first part of the experimentation, the two pressure ratios 2.96 and 3.947 as shown in Table 2. were chosen in Lahore, Pakistan and the effect of these pressure ratios was calculated on the COPs of the VT Refrigerator on different streams of the air. The value of the COPs increases as the cold mass fraction is increased. Represented in Figures 4,5,6, 7.

(5) In the second part of the experimentations, experiments were performed in the Murree Pakistan. The same inlet pressures, 300kpa and 400kpa, were taken but the pressure ratios were (3.46,4.61) different than those in Lahore Pakistan. This is because Murree is relatively higher in altitude as compared to that of Lahore. For the same inlet Pressures 300kpa and 400kpa the pressure ratios increased in the Murree, the COP decreases as shown in Tables 4 and 5.

(6) Lastly, we have compared the COPs for the same pressures 300kpa and 400kpa as shown in figures 8 and 9 in the Murree and Lahore.

5. Acknowledgment

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input is increased. This is true per their formulas.(3) The Work input is increased because the compressor now must do more work and the COP is decreased because it is the ratio between the refrigerating effect and the work input.

REFERENCES

1. Pinar, A. M., Uluer, O. & Kirmaci, V. Optimization of counter flow Ranque-Hilsch vortex tube performance using Taguchi method. *Int. J. Refrig.* **32**, 1487–1494 (2009).
2. Xue, Y., Arjomandi, M. & Kelso, R. The working principle of a vortex tube. *Int. J. Refrig.* **36**, 1730–1740 (2013).
3. Xue, Y., Arjomandi, M. & Kelso, R. Experimental study of the thermal separation in a vortex tube. *Exp. Therm. Fluid Sci.* **46**, 175–182 (2013).
4. Stephan, K., Lin, S., Durst, M., Huang, F. & Seher, D. An investigation of energy separation in a vortex tube. *Int. J. Heat Mass Transf.* **26**, 341–348 (1983).
5. Nimbalkar, S. U. & Muller, M. R. An experimental investigation of the optimum geometry for the cold end orifice of a vortex tube. *Appl. Therm. Eng.* **29**, 509–514 (2009).
6. Winkelmanns, G., Cocle, R., Dufresne, L. & Capart, R. Vortex methods and their application to trailing wake vortex simulations. *Comptes Rendus Phys.* **6**, 467–486 (2005).
7. Nellis, G. F. & Klein, S. A. the Application of Vortex Tubes To Refrigeration. in *Cycle 8* (1962).
8. Wu, Y. T., Ding, Y., Ji, Y. B., Ma, C. F. & Ge, M. C. Modification and experimental research on vortex tube. *Int. J. Refrig.* **30**, 1042–1049 (2007).
9. Rattanongphisat, W., Riffat, S. B. & Gan, G. Thermal separation flow characteristic in a vortex tube: CFD model. *Int. J. Low-Carbon Technol.* **3**, 282–295 (2008).
10. Eiamsa-ard, S. & Promvonge, P. Review of Ranque-Hilsch effects in vortex tubes. *Renewable and Sustainable Energy Reviews* **12**, 1822–1842 (2008).

11. Simões-Moreira, J. R. An air-standard cycle and a thermodynamic perspective on operational limits of Ranque-Hilsh or vortex tubes. *Int. J. Refrig.* **33**, 765–773 (2010).
12. Noor, D. Z., Mirmanto, H., Sarsetiyanto, J., Soedjono, D. M. E. & Setyawati, S. B. Numerical Study of Flow and Thermal Field on a Parallel Flow Vortex Tube. *Engineering* **04**, 774–777 (2012).
13. Perry, R.H., Green, D. W. *Perry's Chemical Engineers' Handbook* (8th rev. ed.). *Perrys' chemical engineers' handbook* (2004). doi:10.1036/0071511245
14. Devade, K. D. & Pise, A. T. Effect of Mach number, valve angle and length to diameter ratio on thermal performance in flow of air through Ranque Hilsch vortex tube. *Heat Mass Transf. und Stoffuebertragung* **53**, 161–168 (2017).
15. Carrascal, E. & Sala Lizarraga, J. M. Mass, energy, entropy and exergy rate balance in a ranque-hilsh vortex tube. *J. Technol. Sci. Educ.* **3**, (2013).
16. Xue, Y., Arjomandi, M. & Kelso, R. A critical review of temperature separation in a vortex tube. *Exp. Therm. Fluid Sci.* **34**, 1367–1374 (2010).
17. Nitsche, M. & Krasny, R. A numerical study of vortex ring formation at the edge of a circular tube. *J. Fluid Mech.* **276**, 139–161 (1994).
18. Karthikeya Sharma, T., Amba Prasad Rao, G. & Madhu Murthy, K. Numerical Analysis of a Vortex Tube: A Review. *Arch. Comput. Methods Eng.* **24**, 251–280 (2017).
19. Marques, C. H., Isoldi, L. A., Dos Santos, E. D. & Rocha, L. A. O. Constructal Design of a Vortex Tube for Several Inlet Stagnation Pressures. *Rev. Eng. Térmica* **11**, 85 (2018).
20. Ahlborn, B., Camire, J. & Keller, J. U. Low-pressure vortex tubes. *J. Phys. D. Appl. Phys.* **29**, 1469–1472 (1996).

