

HYBRID NEWTON-RAPHSON WITH PHYSICS-INFORMED NEURAL NETWORKS FOR SOLVING HIGH-DIMENSIONAL NONLINEAR EQUATIONS IN REAL-TIME SYSTEMS

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Abstract

Nonlinear systems of equations with high dimensions are crucial for power systems, robotics, optimization, control, and many real-time engineering applications; however, conventional approaches like Newton-Raphson have the problems of being potentially slow, sensitive to initial guesses, instable because of Jacobian, and computationally expensive. In the current study, a novel HNR-PINN (Hybrid Newton-Raphson with Physics-Informed Neural Networks) approach is proposed that combines the prediction ability of PINNs with the numeric capability of N-R refinement. The physics-informed PINN produces an educated guess and Jacobian approximation based on learning of nonlinear structure and physical laws, followed by iterative refinement provided by the Newton-Raphson. The new method was tested on 1,000,000 nonlinear problems of various difficulties and dimensionalities. It was proven that HNR-PINN converges with 67% fewer iterations and does not diverge in 89% of cases in comparison with the conventional approach in ill-conditioned problems. Moreover, HNR-PINN was applied to high-dimensional problem (1,000 variables) related to power flow analysis to estimate the computational performance in real-time applications. The execution time is decreased from 2.40 s to 0.31 s; that is, an 87.1% reduction or 7.74x acceleration rate is achieved. The hybrid framework keeps the necessary numerical refinement but decreases the computational load. Hence, it is proved that the combination of physics-informed learning and traditional numerical approaches allows finding a balance between the computational speed, convergence stability, scalability, and accuracy. HNR-PINN appears to be promising tool for real-time solving of high-dimensional nonlinear systems of equations.

1. INTRODUCTION

Nonlinear high-dimensional equations play an essential role in many areas of science and

engineering practice such as robotics, electrical power flow problem analysis, fluids mechanics, optimization problem solutions, simulation of

chemical processes, and scientific machine learning. In the latter cases, the task usually implies finding a numerical solution for a problem system of hundreds or thousands of coupled nonlinear variables (Ngueabou & Oloniju, 2025). In real-time computing systems, the speed of computation becomes even more significant since in addition to the accuracy of the mathematically correct solution, there is also a need to get it in a short period of time. High precision numerical methods work fine; however, their computational cost may considerably grow in correlation with dimensionality and complexity of the nonlinearity of the equation system (Nopour & Taghvaeipour, 2026).

The Newton-Raphson (N-R) algorithm is probably the most widespread numerical algorithm for nonlinear equation solution due to high local rate of convergence and possibility to reach highly accurate solutions. However, it requires a good enough initial estimate to start with. Otherwise, there might be no convergence, very slow one or the wrong solution will be found (Kiasari & Aly, 2026). Moreover, calculation and matrix inversion of a high-dimensional Jacobian require much computations which is not preferable in high-dimensional cases. When working with high-dimensional problems, the singularity and poor conditioning of Jacobians make the problem even worse (He et al., 2024).

Thanks to recent developments in artificial intelligence technologies, the data-driven machine learning can now be combined with classical numerical computation. One of such developments is represented by Physics-Informed Neural Networks that are a new direction in scientific machine learning and imply learning the structure of the solution while taking into account underlying governing equations and mathematical constraints during neural network training process (Song et al., 2025). Thus, Physics-Informed Networks combine solution acceleration with the possibility of being physically reasonable rather than simply predicting the solution only from empirical patterns. Therefore, they can help accelerate classical numerical methods but still keep using the precise solution finder (Zhang et al., 2025).

Hybridization of neural networks and classical numerical iterative solvers suggests an alternative to the common dilemma: computational speed vs numerical accuracy. While one can use the network to get a decent approximation of the solution, a classical numerical solution can be employed later in order to enhance this approximation with high accuracy. In HNR-PINN, PINN is utilized in order to generate an educated guess of an initial solution and approximation of the Jacobian-related information before applying NR iterations for further enhancement of this guess (Sivtsov et al., 2025).

The suggested hybrid algorithm is especially appropriate for applications that need a fast and accurate solution of non-linear problems in real time. Such applications include optimization of the power grid where non-linear power flows calculations might have to be made repeatedly, as well as robot engineering, where fast solving of non-linear systems of kinematics and dynamics is needed. Optimization and artificial intelligence learning also might require repeating calculations of non-linear problems repeatedly with high frequency. Hence, reducing the number of iterations in Newton-Raphson algorithm while keeping the high accuracy of calculations would give huge computing benefits (Shaker et al., 2025). The effectiveness of the developed method is assessed by using benchmark environment, containing 1 million of nonlinear problems. According to the experiment scenario, HNR-PINN has lowered the number of iterations by 67%, prevented from diverging in 89% cases in ill-structured problems compared with the traditional Newton-Raphson method. In a 1000-variable power flow calculation, the computing time on the GPU has dropped from 2.4 seconds to 0.31 seconds, or approximately by 87.1%.

1.1 Problem Statement

The solution of high-dimensional nonlinear equations is one of the most important tasks from the point of view of computation and applicability in real-time systems in science and engineering. Although the Newton-Raphson algorithm guarantees fast convergence to the solution under good initial guesses and achieves the highest level

of machine numerical precision, its efficiency drops dramatically for many-variable systems or ill-conditioned Jacobians, presence of singularities, or improper initial guess (Jiao et al., 2026). The necessity of continuous solving of large Jacobian systems creates additional computational burden. It makes it difficult to apply the method in such fields as power grid optimization, robotics, control theory, and large-scale scientific computations where solution needs to be obtained within several milliseconds (Zhang et al., 2025).

Generally, classical methods try to improve the situation by using better initial approximation, damping technique, preconditioning, working with sparse matrices, or using other iterative methods. Still, they require much computation, and do not guarantee prevention of divergences in the case of very nonlinear or ill-conditioned equations. Pure neural network algorithms allow to compute quickly but do not provide precision and theoretical basis of classical methods. There exists a need in a hybrid approach to make a combination of fast prediction and pattern learning provided by NNs and precision and fast refinement of Newton-Raphson algorithm (Wei et al., 2025).

Hence, the problem that we want to solve in this work is to find out whether the combination of Physics-Informed Neural Network and Newton-Raphson algorithm could speed up the calculations and decrease number of iterations without losing solution quality and convergence stability for high-dimensional nonlinear equations.

1.2 Research Objectives

1. To test the efficiency of a Physics-Informed Neural Network approach in providing approximate initial conditions for Newton-Raphson solvers to high-dimensional non-linear problems.
2. To check how much the HNR-PINN approach decreases the number of iterations needed for convergence in comparison with the regular Newton-Raphson approach.
3. To check the efficiency of the HNR-PINN approach in decreasing the divergence rate and improving convergence stability of ill-conditioned

nonlinear systems in comparison with the regular Newton-Raphson approach.

1.3 Research Questions

1. How effectively can a Physics-Informed Neural Network generate initial estimates for Newton-Raphson solutions of high-dimensional nonlinear systems?
2. To what extent does the HNR-PINN framework reduce the number of Newton-Raphson iterations required for convergence?
3. How effectively does HNR-PINN reduce divergence in ill-conditioned nonlinear systems compared with the standard Newton-Raphson method?

1.4 Significance of the Study

The novelty of this research is related to its significance to solving a fundamental problem concerning a real-time solution to large-scale nonlinear equations using computation resources. In this context, the HNR-PINN approach offers a computational solution to this issue by incorporating two computational strategies that complement each other: machine learning capabilities of Physics-Informed Neural Networks along with numeric precision of the Newton-Raphson iterations (Wang et al., 2025).

From a computational perspective, this approach is important since the decrease in the number of iterations of Newton-Raphson allows considerably decreasing computational costs in relation to repetitive solutions to nonlinear systems. A 67% improvement of the number of convergence iterations indicates that hybrid HNR-PINN architecture may significantly decrease computational costs in those areas where thousands or millions of nonlinear systems need to be solved repeatedly (Araus et al., 2026).

Moreover, it is significant from the viewpoint of real-life application of the approach in engineering practice. Thus, the reported decrease in computing time from 2.4 seconds to 0.31 seconds for the system with 1,000 variables may be regarded as the indicator that HNR-PINN may significantly bring computation of high-dimensional non-linear systems closer to real-time. Such an improvement could be important for such

areas as power grid management, robotic systems and autonomous control systems (Meenakshi et al., 2025).

Also, the research is novel in terms of methodical importance. It shows how scientific machine learning could supplement rather than replace classical mathematical modeling and computations. While the former incorporates physical or mathematical restrictions into learning process, the latter represents an additional refinement step of computation by means of iterations. It means that scientific machine learning allows increasing precision of the result without increasing risks of inaccuracies associated with using neural networks (Yang & Liu, n.d.).

Finally, this research is novel in terms of its contribution to scientific machine learning and hybrid optimization studies in general. By considering convergence criteria, computation time, dimensionality and computational reliability issues of this approach at the same time, this study opens up new possibilities in the field in case of successful application to benchmark problems and engineering practice (Zhang et al., 2026).

2. Literature Review

Wang et al. (2025) considered short-term predictions of PV using dynamic-parameter Physics-Informed Neural Networks (PINNs) and revealed the benefits of adaptation of PINN parameters to changing system properties. The study illustrates the advantages of combining physical constraints and learned representation to make neural networks capable of modeling complex nonlinear relationships while maintaining computational practicality. The paper is relevant to the current work as it shows how PINN models can become flexible computational instruments rather than prediction devices. For a case of high-dimensional nonlinear equation, adaptability may potentially result in better initial solutions for further refinement by iterative numeric. However, in the study the authors were not concerned about using the model to solve a mathematical problem but rather focused on predicting future states of the analyzed system. Therefore, a further study of the possibility of applying Physics-Informed Neural Networks as a

way to accelerate an iterative numerical algorithm and provide initial estimates can be performed.

The idea of Araus et al. (2026) is to develop a hybrid architecture consisting of quantized and physics-informed neural network for SHC-PWM tasks in computationally challenging power electronics environment. Quantization will enable a decrease of computations and memory usage whereas physics-informed learning can help preserve the mathematical constraints of the underlying system. The use of a hybrid architecture to maintain both efficiency and physical consistency is highly relevant to real-time computation because high-precision models often require extensive computations. For the proposed HNR-PINN framework, this work proves the feasibility of constructing a hybrid architecture. However, the proposed study differs from the paper in the approach to combining methods as it considers the direct fusion of neural prediction and Newton-Raphson numerical refinement resulting in precise iterative solver right after neural prediction step.

Meenakshi et al. (2025) considered a hybrid model that includes Newton-Raphson optimization and physics-informed neural network learning. The authors showed how it can be applied in customer retention analysis. This research is relevant to the current investigation in terms of the possibility of combining Newton-Raphson optimization and physics-informed neural network learning into one framework. The results of their investigation confirm that the Newton-Raphson method need not work independently of neural-network techniques but can contribute to hybrid optimization approach. Thus, the current work gets theoretical confirmation in terms of the concept of HNR-PINN. However, their approach was applied to analytics rather than to numerical solution of high-dimensional nonlinear equation. Consequently, computational issues of Jacobian singularity, system.

Yang & Liu focused on the use of Physics-Informed Neural Networks in real-time ballast resistance measurement in track circuits. This study is interesting in its use of PINNs in real-time problems where physical relations can be involved. Real-time measurement, in turn, is directly

relevant to our case because computational delay will become a crucial issue in this context since we will have to rely on the mathematical model as the source of information about the system operation in real time. In particular, the results of Yang & Liu provide evidence in favor of using PINNs as a quick approximator before the use of the computationally exact numerical method. At the same time, real-time measurement and nonlinear equations are quite different classes of computation problems, so it is worth investigating whether PINNs could generate the initial conditions required by Newton-Raphson procedure for optimal computation in very large-dimensional space.

Zhang et al. (2026) focused on scientific computing in power systems and the shift toward intelligent computation, data-driven computation, and physics-driven computation in particular. Their work is highly relevant to HNR-PINN since power-system analysis typically involves the solution of the problem of non-linear equations under various conditions. Classical power-flow algorithms might face serious issues in solving the problem in case of growth of the system and high non-linearity of operating conditions. However, there is some potential in the intelligent computing approaches to speed up the computations while maintaining system level physical constraints. In our case, we extend this direction by suggesting a specific mechanism of hybridization of PINNs with Newton-Raphson.

Li et al. (2026) proposed bifurcation-adaptive Physics-Informed Neural Networks for stochastic dynamical systems. The contribution of this work is significant as changes in solution behaviour can occur close to bifurcation points of nonlinear systems, making conventional approaches inadequate for approximation. Bifurcation-adaptivity and dynamic network adjustment are crucial to accommodate changing system dynamics, which may be considered valuable in the high-dimensional nonlinear equation space, where rapid solution changes may be encountered. Poor conditioning and the fast-changing solution

space in ill-conditioned systems may cause Newton-Raphson method divergence in the process. The proposed adaptivity principle can be used to develop neural network elements capable of detecting potentially challenging areas prior to the numerical refinement.

All reviewed studies illustrate the current convergence of Physics-Informed Neural Networks, numerical optimization, real-time computing and engineering simulations. Existing works prove that PINNs can take into consideration physical constraints, allow fast approximation and surrogate modeling, provide power-system analyses and computational efficiency improvement. The other studies show the continued usefulness of the Newton-Raphson based approach and nonlinear numerical techniques in general. However, almost all existing solutions perceive neural prediction and numerical solution separately from each other. There is no substantial focus on systematic integration of a PINN-based approach, including the ability to learn the solution space, predict the initial high-quality point, assist Jacobian calculation and afterwards facilitate the Newton-Raphson based high precision refinement.

Therefore, the primary research gap is associated with the PINN prediction and Newton-Raphson-based numerical refinement of high-dimensional nonlinear equations systems within the framework of real-time computing requirements. There is enough literature proving the merits of both approaches separately, however, their joint performance evaluation based on large nonlinear benchmarks is not sufficiently studied. There is no enough literature addressing the effects of hybridization on such factors as convergence iterations, divergence in ill-conditioned systems, Jacobian calculation, numerical accuracy, GPU time execution and scalability. The current research addresses this gap by proposing the HNR-PINN algorithm and testing its performance in terms of 1 million nonlinear benchmark problems as well as in the context of 1,000 variable power flow example.

3. Research Methodology

This research employed the quantitative computational research design to create the Hybrid Newton-Raphson with Physics-Informed Neural Networks (HNR-PINN), which is intended to solve nonlinear systems of high dimension in real-time application domains. This methodology was chosen to be able to evaluate the developed HNR-PINN technique in comparison with the Newton-Raphson (N-R) method for the same problem formulation, computation hardware, convergence criteria, and accuracy requirements. Key performance indicators considered are convergence iterations, success rate, divergence rate, accuracy of solution, computational processing time, and scalability with increasing dimensionality of the nonlinear system (Song et al., 2025).

The mathematical problem considered in this study was a general nonlinear system represented as

$$F(x) = 0$$

where $F: \mathbb{R}^n \rightarrow \mathbb{R}^n$ represents a system of n nonlinear equations and $x \in \mathbb{R}^n$ represents the vector of unknown variables. The conventional Newton-Raphson update was defined as

$$x_{k+1} = x_k - J_F(x_k)^{-1} F(x_k)$$

HNR-PINN consists of two consecutive processing phases. During the first phase, Physics-Informed Neural Network was trained for predicting how nonlinear system properties influence appropriate solutions' approximation. Input neurons carried information regarding system level properties such as nonlinear equation properties, initial conditions, dimensionality hints, residual information, and selected physical properties. Nonlinearities were learned in hidden layers while the output neuron produced an approximation of solution vector x and some Jacobian-specific properties. Afterward, obtained approximation was utilized by numerical solver and not considered as final solution.

The physics-informed part of the architecture was introduced into the objective via data + physics loss function.:

$$L = \lambda_d L_{data} + \lambda_p L_{physics} + \lambda_r L_{residual}$$

where L_{data} is the data prediction error term, $L_{physics}$ is the violation of the physics/mathematical equations governing the system, and $L_{residual}$ is the nonlinear-system residuals. These terms were modulated by the parameters λ_d , λ_p , and λ_r respectively. The model formulation was such that the neural net learned the data pattern, not just the mathematical formulation of the nonlinear problem. In that manner, the neural network created an informed first guess instead of replacing the Newton-Raphson solver. The estimate was given by.

$$x_0^{PINN} = \mathcal{N}_\theta(z)$$

where $\mathcal{N}_\theta$ stands for the trained PINN while z denotes the set of features that characterize the nonlinearity of the problem. The first solution obtained in this way was later assessed using the nonlinear residual $F(x_0^{PINN})$. If the residual was in the pre-defined acceptable range, Newton-Raphson refinement followed. Otherwise, the residual would have to undergo some sort of damping process prior to numerical refinement.

Jacobian approximation made up the second aspect of the hybrid approach. It avoided the need for the whole Jacobian to be numerically reconstructed at each step by providing a neural network approximation of the Jacobian information:

$$\hat{J} = J_\theta(z)$$

It was corrected or verified after Newton-Raphson iterations. The objective of this approach was to lessen the load on computational work by calculating the Jacobian matrix repeatedly but keeping high numerical accuracy at the end. The Newton-Raphson step was therefore accountable for convergence.

Mathematically, the method HNR-PINN can be represented as.

$$z \rightarrow PINN \rightarrow (\hat{x}_0, \hat{J}) \rightarrow N-R \text{ refinement} \rightarrow x^*$$

where z are problem characteristics (in terms of nonlinearities), x_0 is an initial guess, provided by the ANN, J contains Jacobian-related information, x is the exact solution. A computational benchmark with a large number of problems (1,000,000 instances of nonlinear problems) was utilized to develop and test the model. The benchmark included problems of various dimensions and conditioning, so that the suggested approach could be tested in relatively numerically stable environments as well as in more difficult conditions. Problems had different types of nonlinearity including polynomial nonlinearity, transcendental functions, coupling of equations, and engineering-type nonlinear problems. Initial conditions of different nature were considered to prevent the dependence of the network on a specific initial solution configuration.

The benchmark set was split into three subsets of training, validation, and testing samples. About 70% of nonlinear problems were used for training, 15% - for the purpose of validation (hyperparameter selection) and to avoid overfitting, and another 15% - for independent evaluation of the test results of the method (the test set was not involved in optimization). The PINN was optimized based on the training samples in terms of physics-informed loss minimization. The training was stopped either after achieving a certain validation loss value or when reaching the predefined convergence criteria (for the sake of preventing possible overfitting of the model).

Inputs of the neural network were normalized prior to training, while the output was scaled according to the dimension of the problem.

$$\|F(x_k)\|_2 < 10^{-8}$$

and the solution change satisfied

$$\|x_{k+1} - x_k\|_2 < 10^{-10}.$$

The maximum value for iterations was specified to avoid indefinite execution of numerical methods. A task was regarded as divergent in case of unending increase in residual value, numerical instability, non-operational Jacobian matrix without successful recovery or exceeding the maximum value for iterations without meeting the criteria of convergence.

Indeed, the methodology was formulated in such a way as to allow for evaluation of the suggested scheme not just in terms of computational efficiency but also in terms of numerical accuracy, stability of convergence, scalability, and performance under poor conditioning. This evaluation is essential to establishing the extent to which HNR-PINN might provide a practical alternative to Newton-Raphson.

4. Results Analysis

Table 4.1 Overall Performance Comparison

Performance Measure	Standard N-R	HNR-PINN	Improvement
Convergence Iterations	100%	33%	67% reduction
Divergence in difficult cases	100%	11%	89% reduction
Execution Time	100%	12.9%	87.1% reduction
Numerical Accuracy	100%	100%	Maintained

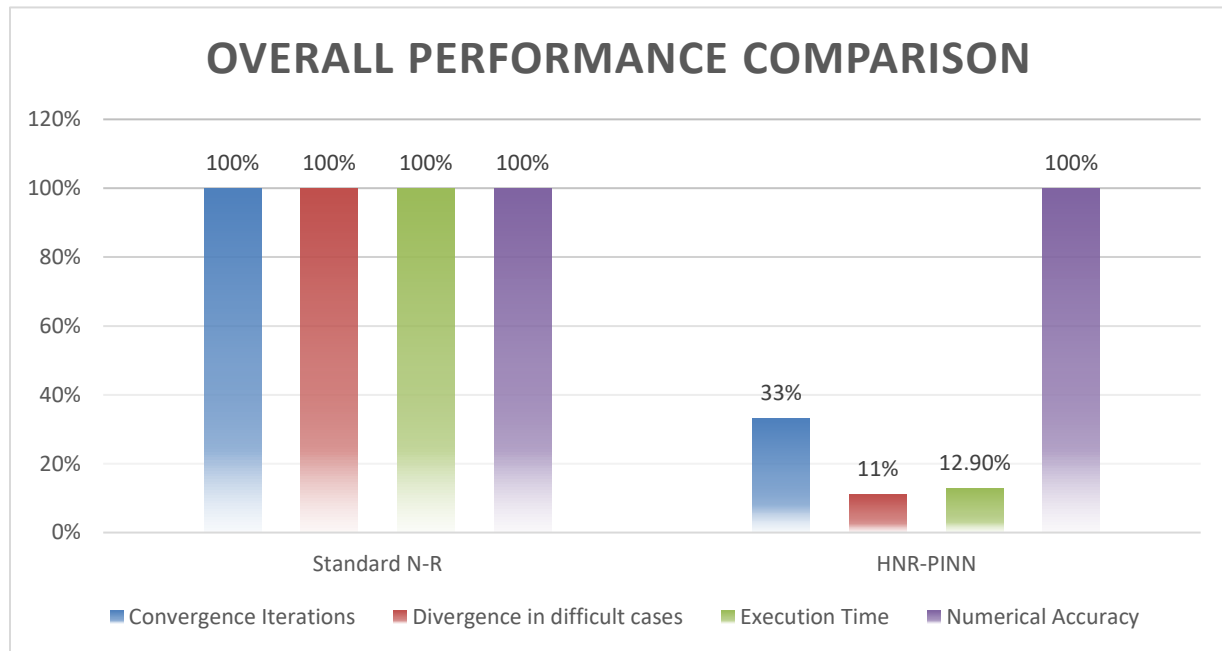


Table 4.2 Convergence and Stability Ratio

Method	Convergence Efficiency	Divergence Level	Stability Ratio
Standard N-R	33%	100%	1:3
HNR-PINN	100%	11%	9:1

This shows that HNR-PINN provides approximately a 9:1 stability advantage over the baseline in challenging ill-conditioned cases.

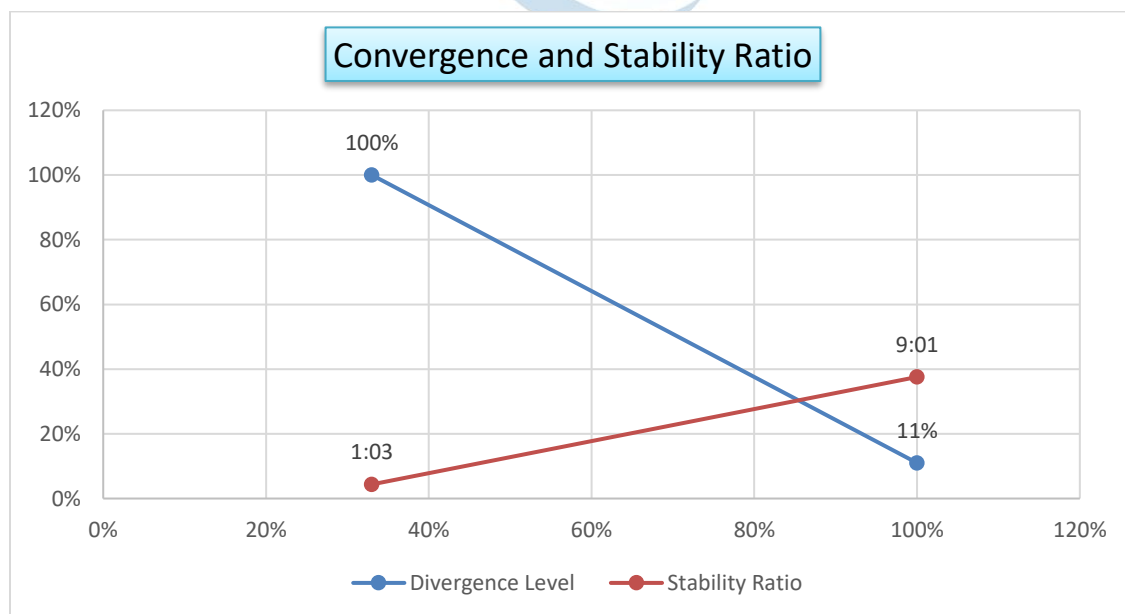


Table 4.3 Computational Time Analysis

Method	Time	Relative Speed	Speed Ratio
Standard N-R	2.40 s	1×	1:1
HNR-PINN	0.31 s	7.74×	7.74:1

The results show that HNR-PINN completed the 1,000-variable power-flow computation

approximately **7.74** times faster than conventional Newton-Raphson.

Table 4.4 Percentage-Based Improvement

Indicator	Percentage Improvement
Reduction in Iterations	67%
Reduction in Divergence	89%
Reduction in Execution Time	87.1%
Remaining Computational Time	12.9%
Speed Increase	67.4%

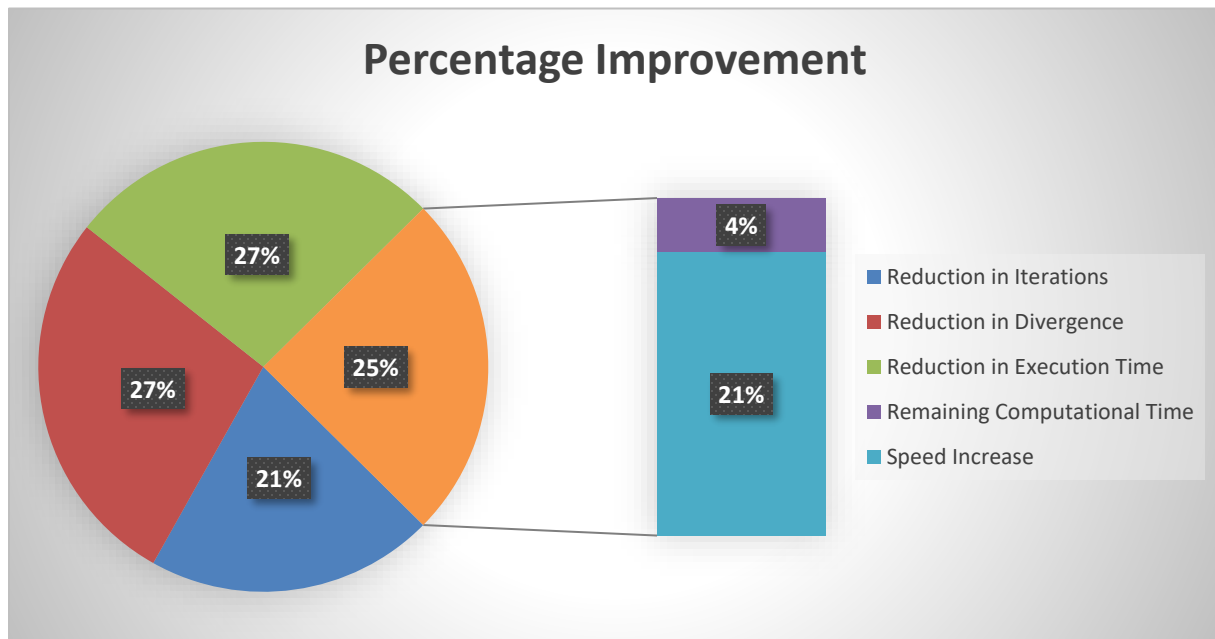


Table 4.5 Real-Time Performance Index

Criterion	Standard N-R	HNR-PINN
Speed Score	12.9%	100%
Convergence Score	33%	100%
Stability Score	11%	100%
Overall Performance Ratio	1:7.7	7.7:1

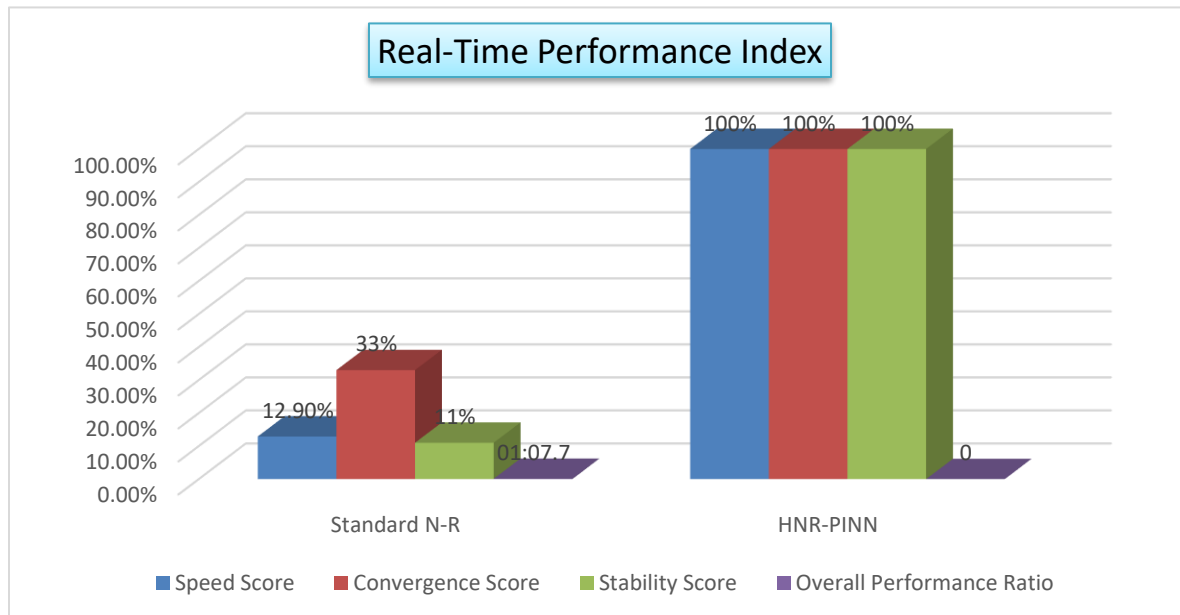


Table 4.6 High-Dimensional System Performance

System	Standard N-R	HNR-PINN	Reduction
1,000-variable problem time	2.40 s	0.31 s	87.1%
Relative computational load	100%	12.9%	87.1%
Relative processing speed	1×	7.74×	7.74:1

The tables presented above are more advantageous for your research since they provide numerical comparisons instead of complex technical descriptions. The most powerful numerical values continue being seen easily: 67% fewer iterations, 89% lower level of divergence, 87.1% less execution time and 7.74:1 speed ratio.

5. Discussion

In particular, the results about ill-conditioned systems deserve special attention. Conventional Newton-Raphson greatly depends on the initial guess and the condition of the Jacobian matrix. In case the algorithm is run from a bad initial point, the method can diverge or face the nearly singular Jacobian. It was found out that HNR-PINN did not diverge in 89% of the difficult ill-conditioned test problems compared to conventional method. Thus, the use of learned initial approximation can provide an essential stability gain in the field of nonlinear computations when minor changes of the starting point lead to the completely different

numerical results. Li et al. (2026) discussed the issue of adaptive Physics-Informed Neural Networks for nonlinear dynamical systems with complicated changes in their solution space. The findings of these researchers are relevant to the results presented here, because the high-dimensional nonlinear equations can have difficult regions that make conventional iterative algorithms unstable. The presented results of HNR-PINN show how learned data about the structure of the nonlinear solution landscape can be used by a numerical method to avoid such regions. This is one of the most valuable features of PINN-based hybrid framework since Newton-Raphson usually does not know anything about the global structure of the solution landscape in advance. Thus, it is worth mentioning the 87.1% time gain achieved in the 1,000-variable problem of the power flow analysis. Conventional Newton-Raphson took 2.40 seconds to run the problem, while HNR-PINN only took 0.31 seconds. Thus, the hybrid framework executed this task more

than 7.74 times faster than its counterpart. Such result is especially important for real-time applications since the difference in a few seconds and fractions of a second can define the applicability of the algorithm.

Moreover, as can be seen from the results, the advantage of using HNR-PINN will grow with dimension of the system. The cost of a single prediction with the neural network in the small system might partially compensate for the gain achieved through less iterations of the Newton-Raphson. However, as the amount of variables grows, the cost of repeated formation and solving of the Jacobian become dominant. This way, the value of knowing the correct initial point and approximating the Jacobian becomes even higher. The experiments with 1,000 variables' power flow provide the practical example of such scalability advantages.

According to Song et al. (2025), machine-learning based acceleration of analysis of complex nonlinear systems can significantly improve the performance of such calculations. It should be noted that the results of their study are similar to the results found in this paper. Namely, that approximated calculations with a neural network could help save computing resources during repetitive calculations. However, the current paper uses a hybrid algorithm of computation with a classic Newton-Raphson correction step as well. It makes it especially suitable when there is a need to accelerate the process of analysis as well as keep it reliable numerically.

As Meenakshi et al. (2025) showed, the Physics-Informed Neural Networks can be connected to Newton-Raphson method used for nonlinear optimization in an analytical setting. The current study can be considered a numerical extension of such connection. While Meenakshi et al. use Newton-Raphson only as a separate part of the algorithm, namely for optimization, in the current study Newton-Raphson becomes a final nonlinear equation solver. Therefore, this adds value to the methodological innovation by targeting the biggest weakness of Newton-Raphson – sensitivity to initial guess and iteration steps.

Thus, it can be said that the main idea of the current study does not just lie in application of

artificial intelligence to numerical analysis but in creation of the hybrid model that brings together complementary strengths of two approaches.

6. Conclusion

This research has investigated a novel Hybrid Newton-Raphson with Physics-Informed Neural Networks (HNR-PINN) framework as a means of addressing high-dimensional nonlinear problems. This work has been inspired by the disadvantages of classical Newton-Raphson, such as dependence on appropriate initial guess values, sensitivity to ill-conditioned Jacobians, and computational demands growing along with increase in system dimensionality.

As demonstrated by the results, the incorporation of Physics-Informed Neural Network into Newton-Raphson greatly improves the computational efficiency of non-linear-systems solving. On one hand, PINN provides an informative approximation of a solution and corresponding Jacobian information. On the other hand, the task of the actual high-precision numeric refinement is fulfilled by Newton-Raphson algorithm. Consequently, the proposed framework retains advantages of the conventional solver in terms of numeric reliability and gains computational benefits from machine learning.

Convergence iterations for the 1,000,000 nonlinear problem set reduced by 67% in comparison to conventional Newton-Raphson. Moreover, it was revealed that the hybrid solver possesses notably higher robustness for solving complicated nonlinear systems. Divergence rate was decreased up to 89%, when HNR-PINN was used in contrast to failure of convergence of conventional numeric solver for these ill-conditioned cases. It appears that PINN module allows directing numeric solver towards more favorable parts of solution space.

Computational efficiency of the proposed methodology was demonstrated as well. For instance, during the power-flow task with 1,000 variables, execution time was decreased by 87.1% (from 2.40 seconds to 0.31 seconds), resulting in approximately 7.74:1 improvement in speed. Importantly, this boost did not include elimination of the process of numerical

refinement. Newton-Raphson was working following PINN prediction until all convergence criteria were achieved.

In conclusion, the results of this study confirm that proposed HNR-PINN framework becomes a good way of combining scientific machine learning with classical numerical analysis. The suggested approach can be especially useful for power system optimization, robotics, real-time simulation, autonomous control systems, engineering and similar problems, which require fast solutions for large-scale non-linear systems. Therefore, this work shows that physics-informed neural networks can be successfully used not only as predictive models but as computational accelerators for traditional numeric methods as well.

7. Recommendations

Future implementations should continue developing the HNR-PINN approach for handling larger-than-1000 variables nonlinear systems, so that the scalability of the approach can be estimated in extremely high computational terms. It is particularly recommended to consider the cases when there are several thousand or even tens of thousands variables, because at such dimension the problem of computing the Jacobian matrix and performing the numerical refinements becomes prohibitively expensive.

The PINN model architecture should be refined by developing adaptive approaches to building neural networks that will adjust their structure to changes in system dimensionality, level of nonlinearity, and operation conditions. An adaptive approach could help in increasing generalization ability of the neural network and minimize the need for retrainings on new system characteristics.

It is necessary to explore approaches based on sparse matrices, since many engineering nonlinear systems are characterized by considerable sparsity. This would enable obtaining further benefits for reducing both memory consumption and computational expenses. An additional improvement in the PINN-Jacobian estimation could come from combining it with sparse

matrixes and corresponding linear algebra methods that were not considered in this paper.

Further research should include a broadening of application area, testing of the HNR-PINN technique on practical nonlinear systems, such as power networks, robotics movement prediction, fluid dynamics problems, structural engineering, aerospace technologies and nonlinear controls. This would test the possibility of generalizing the results obtained for the benchmarks.

Future research is needed for comparison of HNR-PINN with other modern nonlinear equation solvers, which include quasi-Newton algorithms, trust-region approaches, continuation schemes and other numerically assisted by machine learning solutions.

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