

DYNAMIC-SENSITIVITY-GUIDED TRAJECTORY OPTIMIZATION OF A REDUNDANT 7-DOF MANIPULATOR UNDER DYNAMIC-PARAMETER UNCERTAINTY

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Abstract

Dynamic-parameter uncertainty can significantly affect the torque response and robustness of redundant robotic manipulators, particularly when trajectory generation does not account for variations in inertial properties. This study proposes a dynamic-sensitivity-guided trajectory optimization framework for a redundant 7-DOF Franka Emika Panda manipulator. The proposed method first evaluates the sensitivity of the manipulator torque response to 49 dynamic parameters, comprising seven link masses and 42 inertia terms. The sensitivity analysis identifies five dominant link masses that account for 98.67% of the total mean sensitivity and are subsequently used to construct a Dynamic Uncertainty Sensitivity Index (DUSI). Redundancy is exploited through a null-space coordinate represented by a cubic B-spline, allowing the joint trajectory to be modified while maintaining the desired Cartesian motion. The trajectory is optimized using a constrained nonlinear programming formulation that minimizes the normalized mean DUSI and jerk, while imposing limits on the DUSI upper tail, mechanical work, joint motion, and Cartesian position error. For the nominal 5-s trajectory, the optimized solution reduces the mean DUSI by 0.129% and RMS jerk by 3.34%, with a 3.36% increase in absolute mechanical work. The DUSI 95th percentile increases by only 0.007%, remaining within the imposed tolerance. Monte Carlo validation using 1000 paired realizations of $\pm 5\%$ dynamic-parameter uncertainty shows a 0.197% reduction in mean RMS relative torque deviation, with the optimized trajectory performing better in 91.7% of paired trials. The results demonstrate that dynamic sensitivity can provide a quantitative basis for exploiting redundancy to obtain small but statistically consistent improvements in trajectory robustness.

1. Introduction

Redundant robotic manipulators possess more degrees of freedom than are required to perform a specified Cartesian task. This redundancy

provides additional freedom for avoiding joint limits, improving manipulability, avoiding undesirable configurations, and optimizing secondary performance objectives while

maintaining the primary task [1], [2], [3]. Recent trajectory-planning studies have also demonstrated that redundant degrees of freedom can be incorporated into optimization to improve motion quality while satisfying task and joint constraints [4], [5], [6]. However, conventional redundancy-resolution methods mainly exploit this freedom through kinematic or geometric criteria, while the dynamic effect of the selected redundant configuration remains less explored.

The dynamic response of a manipulator depends on its mass distribution, including link masses and inertia parameters. These parameters may differ from their nominal values because of manufacturing tolerances, simplified CAD models, payload variation, parameter-identification errors, and modeling assumptions [7], [8], [9]. Recent studies on robot dynamic identification and uncertainty analysis have shown that uncertainty in physical parameters can influence the predicted dynamic response and trajectory performance [10], [11], [12]. Consequently, a trajectory designed using nominal parameters may produce different actuator torques when the actual dynamic parameters deviate from the assumed model.

A direct approach to this problem is to include uncertainty in trajectory optimization. However, not all dynamic parameters have the same influence on the torque response. A small relative perturbation in one parameter may produce a substantially larger torque variation than the same perturbation in another parameter. Recent reliability-based studies of industrial robots have demonstrated that sensitivity analysis can identify a small subset of critical uncertain parameters from a larger parameter set [13], [14], [15]. Treating all uncertain parameters equally can therefore obscure the parameters that actually dominate the dynamic response and can increase the computational burden of subsequent optimization.

Sensitivity analysis provides a systematic way to address this issue. By perturbing individual dynamic parameters and measuring the resulting torque variation, the parameters can be ranked according to their influence on the manipulator

dynamics. Such a ranking can subsequently be used to construct a sensitivity-weighted uncertainty metric. Recent research has demonstrated that sensitivity-aware trajectory planning can improve robustness of redundant manipulation under payload uncertainty [16]. However, the existing approach primarily considers closed-loop sensitivity associated with payload variations, leaving an opportunity to investigate parameter-wise sensitivity of the inverse-dynamics torque response and its direct use in redundant trajectory optimization.

Redundant manipulators provide an additional mechanism for reducing this sensitivity. A null-space motion can change the internal joint configuration while preserving the desired Cartesian motion. Because the inertia, Coriolis/centrifugal, and gravity terms depend on the manipulator configuration, redundant motion can also modify the torque response and its sensitivity to uncertain dynamic parameters [17], [18], [19]. Recent work has further shown that null-space selection can influence the dynamic response of redundant manipulators. This indicates that redundancy can be considered not only as a kinematic resource but also as a mechanism for modifying dynamic behavior.

This observation motivates the approach proposed in this study. Instead of treating redundancy solely as a kinematic resource, the redundant degree of freedom is used as a design variable for dynamic robustness. The proposed framework first identifies the dynamic parameters that dominate the torque sensitivity, constructs a Dynamic Uncertainty Sensitivity Index (DUSI), evaluates the local sensitivity of DUSI along the null-space direction, and then optimizes a smooth redundant trajectory.

The study considers the 7-DOF Franka Emika Panda manipulator. The initial sensitivity analysis considers 49 dynamic parameters, consisting of seven link masses and 42 inertia components. The dominant parameters are subsequently used to construct the DUSI. A cubic B-spline is employed to parameterize the redundant motion, and the resulting control points are optimized using constrained nonlinear programming. Spline-based

and constrained optimization approaches have recently been applied to redundant-manipulator trajectory generation to obtain smooth trajectories while satisfying motion requirements [20], [21].

The optimization minimizes the normalized mean DUSI while including RMS jerk as a secondary smoothness term. An upper-tail DUSI constraint is imposed to prevent deterioration of the high-sensitivity portion of the trajectory. Joint limits, joint velocity and acceleration limits, redundant displacement, Cartesian deviation, and mechanical work are also constrained. Unlike conventional trajectory optimization based primarily on time, energy, manipulability, or smoothness, the proposed formulation uses dynamic sensitivity to define the principal robustness objective.

Finally, the optimized trajectory is evaluated independently using paired Monte Carlo simulations. The uncertainty validation uses 1000 realizations of dynamic-parameter perturbations within $\pm 5\%$. The same uncertainty realization is applied to the nominal and optimized trajectories, allowing a direct paired comparison. Recent reliability studies have demonstrated the value of statistical uncertainty propagation for evaluating robot trajectory performance under uncertain parameters [22], [23]. The separation between deterministic optimization and independent uncertainty validation also allows the robustness benefit to be assessed without using the validation realizations directly during optimization.

The contributions of this work are threefold. First, a parameter-wise dynamic sensitivity analysis is used to identify the dominant dynamic parameters of a redundant manipulator. Second, this sensitivity information is incorporated into a DUSI-based null-space trajectory optimization framework. Third, the resulting trajectory is independently evaluated under paired dynamic-parameter uncertainty to determine whether the deterministic sensitivity reduction produces a measurable robustness benefit. The remainder of the paper is organized as follows. Section 2 presents the dynamic model and problem formulation. Section 3 describes the proposed sensitivity-guided redundancy optimization

methodology. Section 4 presents the numerical results and their discussion. Section 5 concludes the paper and discusses the limitations and future research directions.

2. Dynamic Model and Problem Formulation

2.1 Manipulator Dynamics

The configuration of the 7-DOF manipulator is represented by

$$q(t) = [q_1(t), q_2(t) \dots q_7(t)]^T, \quad (1)$$

where $q_i(t)$ is the angular position of joint i in rad and t denotes time in seconds. The corresponding joint velocity and acceleration are

$$\dot{q}(t) = [\dot{q}_1(t), \dot{q}_2(t) \dots \dot{q}_7(t)]^T, \quad (2)$$

and

$$\ddot{q}(t) = [\ddot{q}_1(t), \ddot{q}_2(t) \dots \ddot{q}_7(t)]^T, \quad (3)$$

where the units are rad/s and rad/s^2 , respectively.

The rigid-body inverse dynamics are expressed as

$$\tau = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q), \quad (4)$$

where $\tau \in \mathbb{R}^7$ is the joint torque vector in N·m, M is the joint-space inertia matrix, $C\dot{q}$ represents Coriolis and centrifugal torques, and G represents gravitational torques.

For scalar evaluation of the seven joint torques, the torque norm is defined as

$$T(t) = \|\tau(t)\|_2. \quad (5)$$

The resulting torque norm is expressed in N·m and is used as the principal dynamic response in the sensitivity analysis.

2.2 Nominal Trajectory

The nominal trajectory is generated between the prescribed initial and final joint configurations. Its boundary conditions are:

$$q(0) = q_{\text{start}}, q(T_f) = q_{\text{goal}}, \quad (6)$$

where T_f is the trajectory duration.

The nominal position, velocity, and acceleration profiles are denoted by

$$q_0(t), \dot{q}_0(t), \ddot{q}_0(t). \quad (7)$$

The corresponding nominal torque is calculated from the inverse-dynamics model as

$$\tau_0(t) = \tau(q_0(t), \dot{q}_0(t), \ddot{q}_0(t)). \quad (8)$$

The nominal torque norm is consequently

$$T_0(t) = \|\tau_0(t)\|_2. \quad (9)$$

The nominal trajectory used in the study has a duration of 5 s and is initially generated with 501 samples.

2.3 Dynamic-Parameter Set

For each link i , the mass is denoted by m_i and the inertia tensor is represented by

$$I_i = \begin{bmatrix} I_{xx,i} & I_{xy,i} & I_{xz,i} \\ I_{xy,i} & I_{yy,i} & I_{yz,i} \\ I_{xz,i} & I_{yz,i} & I_{zz,i} \end{bmatrix}. \quad (10)$$

The mass has units of kg and the inertia components have units of $\text{kg}\cdot\text{m}^2$. Each symmetric inertia tensor contains six independent components. Thus, the seven links provide 42 independent inertia parameters. Together with the seven link masses, the sensitivity analysis considers 49 scalar dynamic parameters. The complete parameter vector is written as

$$\theta = [\theta_1, \theta_2, \dots, \theta_{49}]^T. \quad (11)$$

2.4 Parameter-Wise Sensitivity

Each dynamic parameter is perturbed independently while all other parameters remain at their nominal values. A relative perturbation of 1% is used.

For parameter θ_p , the positive and negative perturbations are

$$\theta_p^+ = \theta_p(1 + \delta), \quad \theta_p^- = \theta_p(1 - \delta), \quad (12)$$

where δ is the dimensionless relative perturbation. The central finite-difference torque sensitivity is calculated as

$$\frac{\partial \tau}{\partial \theta_p} \approx \frac{\tau_p^+ - \tau_p^-}{2\delta\theta_p}. \quad (13)$$

To remove the influence of parameter units and nominal parameter magnitude, the normalized sensitivity is defined as

$$S_p(t) = \frac{\left\| \frac{\partial \tau}{\partial \theta_p} \right\|_2 |\theta_p|}{T_0(t) + \varepsilon_\tau}, \quad (14)$$

where $S_p(t)$ is dimensionless and ε_τ is a small torque regularization term.

The mean and upper-tail sensitivity are then calculated over the trajectory:

$$\bar{S}_p = \frac{1}{N_t} \sum_{k=1}^{N_t} S_p(t_k), \quad (15)$$

and

$$S_{p,95} = P_{95}[S_p(t_1), \dots, S_p(t_{N_t})]. \quad (16)$$

These measures are used to rank the 49 dynamic parameters.

2.5 Dynamic Uncertainty Sensitivity Index

The sensitivity ranking identifies m_6, m_7, m_5, m_3 , and m_4 as the five dominant parameters. Their normalized sensitivity-derived weights are 0.39327, 0.18261, 0.18219, 0.12252, and 0.11942, respectively.

The DUSI is defined as

$$DUSI(t) = \sum_{j=1}^5 w_j S_j(t), \quad (17)$$

where w_j is the dimensionless weight of dominant parameter j , and $S_j(t)$ is its normalized sensitivity.

The trajectory-level mean DUSI is

$$D\bar{USI} = \frac{1}{N_t} \sum_{k=1}^{N_t} DUSI(t_k), \quad (18)$$

while the upper-tail measure is

$$DUSI_{95} = P_{95}[DUSI(t_1) \dots DUSI(t_{N_t})]. \quad (19)$$

The maximum DUSI is

$$DUSI_{\max} = \max_k DUSI(t_k). \quad (20)$$

These three quantities respectively characterize the average, upper-tail, and worst-case sensitivity of the trajectory.

2.6 Null-Space Motion

The Panda provides one redundant degree of freedom for the selected Cartesian task. The geometric Jacobian is

$$J(q) \in \mathbb{R}^{6 \times 7}. \quad (21)$$

A normalized null-space direction $n(q)$ satisfies

$$J(q)n(q) = 0. \quad (22)$$

The null-space direction is obtained from the right singular vector associated with the smallest singular value of the Jacobian and is normalized to unit Euclidean norm. Its orientation is made continuous along the trajectory to avoid artificial sign reversals.

The local effect of redundancy on DUSI is evaluated by

$$q^+ = q_0 + \alpha n, \quad q^- = q_0 - \alpha n, \quad (23)$$

where α is a small displacement in rad.

The local DUSI sensitivity with respect to the redundant coordinate is

$$\frac{dDUSI}{d\eta} \approx \frac{DUSI(q^+) - DUSI(q^-)}{2\alpha}. \quad (24)$$

A negative value indicates that positive null-space displacement locally reduces DUSI.

2.7 Problem Formulation

The redundant displacement is represented by a cubic B-spline,

$$\eta(t) = \sum_{i=1}^{n_c} B_i(t)c_i, \quad (25)$$

where $B_i(t)$ are dimensionless cubic B-spline basis functions and c_i are control points in rad.

Eight control points are used, with the first and last fixed at zero:

$$c_1 = 0, c_8 = 0. \quad (26)$$

The six internal control points form the optimization vector

$$x = [c_2, c_3, c_4, c_5, c_6, c_7]^T. \quad (27)$$

The optimized joint trajectory is constructed as

$$q_{\text{opt}}(t) = q_0(t) + n(t)\eta(t). \quad (28)$$

The final optimization objective is

$$J(x) = 0.90 \frac{D\bar{USI}(x)}{D\bar{USI}_0} + 0.10 \frac{J_{\text{jerk}}(x)}{J_{\text{jerk},0}} \quad (29)$$

where the first term represents normalized dynamic sensitivity and the second provides trajectory-smoothness regularization.

The absolute mechanical work is calculated from

$$W = \int_0^{T_f} |\tau^T(t)\dot{q}(t)| dt. \quad (30)$$

The optimization is subject to the upper-tail sensitivity constraint

$$DUSI_{95} \leq 1.0001 DUSI_{95,0}, \quad (31)$$

the mechanical-work constraint

$$W \leq 1.05 W_0, \quad (32)$$

and the redundant-motion constraint

$$|\eta(t)| \leq 0.05 \text{ rad}. \quad (33)$$

The optimized trajectory must also satisfy the Panda joint-position, velocity, and acceleration limits and maintain a Cartesian position deviation below 1 mm.

The final optimization problem is therefore

$$\min_x J(x) \quad (34)$$

subject to the constraints defined above.

3. Proposed Methodology

3.1 Sensitivity-Guided Optimization Framework

The proposed framework consists of four computational stages. A nominal trajectory is first generated, and its inverse-dynamics response is calculated. The 49 dynamic parameters are then perturbed individually to establish the sensitivity ranking. The five dominant parameters are used to construct DUSI, after which the local DUSI gradient along the null-space direction is evaluated. Finally, a B-spline representation of the redundant motion is optimized under the specified robustness and trajectory constraints.

The important distinction is that the sensitivity analysis is performed before the redundancy optimization. Therefore, the optimization is informed by the dynamic characteristics of the robot rather than relying on arbitrary uncertainty weights.

3.2 Null-Space Sensitivity Guidance

The local gradient in (24) provides a directional measure of whether redundant motion is favorable or unfavorable from the DUSI perspective. The finite-difference step is selected sufficiently small to characterize local behavior without introducing significant nonlinear displacement effects.

The numerical convergence of this gradient is checked using multiple perturbation amplitudes. Once the direction is verified, the B-spline trajectory provides a smooth global representation of the redundant displacement rather than independently optimizing the redundancy at every trajectory sample.

This distinction is important because independent sample-by-sample optimization could produce discontinuous redundant motion. The B-spline representation imposes smoothness through a small number of control variables.

3.3 B-Spline Redundancy Optimization

The six internal B-spline control points are optimized using constrained nonlinear programming. For each candidate vector x , the redundant displacement is generated, the optimized joint trajectory is calculated, and its velocity and acceleration are obtained numerically.

Inverse dynamics is then used to obtain the torque response.

The candidate trajectory is evaluated using mean DUSI, DUSI-95, mechanical work, RMS jerk, torque norm, joint limits, redundancy amplitude, and Cartesian position deviation.

The objective deliberately gives dominant importance to mean DUSI. RMS jerk contributes only as a regularization term, while DUSI-95 is treated as a hard constraint. This separation prevents the upper tail of the sensitivity distribution from being traded freely against average sensitivity.

3.4 Independent Uncertainty Validation

The optimization does not use Monte Carlo samples. After obtaining the optimized trajectory, an independent uncertainty experiment is performed. For realization r , each dynamic parameter is perturbed according to

$$\theta_p^{(r)} = \theta_p (1 + \xi_p^{(r)}), \quad (35)$$

where

$$\xi_p^{(r)} \sim U[-0.05, 0.05]. \quad (36)$$

The same realization is applied to both the nominal and optimized trajectories. This paired arrangement removes much of the variability that would occur if the two trajectories were tested using different random parameter sets.

The RMS relative torque deviation is calculated for each trajectory and each realization. The resulting paired differences are then analyzed using both parametric and nonparametric statistical tests.

4. Results and Discussion

4.1 Parameter Sensitivity

The parameter-wise analysis shows a strongly nonuniform sensitivity distribution. The five dominant parameters are the masses of links 6, 7, 5, 3, and 4. Link 6 mass is the most influential parameter, with a mean sensitivity of 0.499506 and a 95th-percentile sensitivity of 0.533706. The complete ranking is presented in Fig. 1.

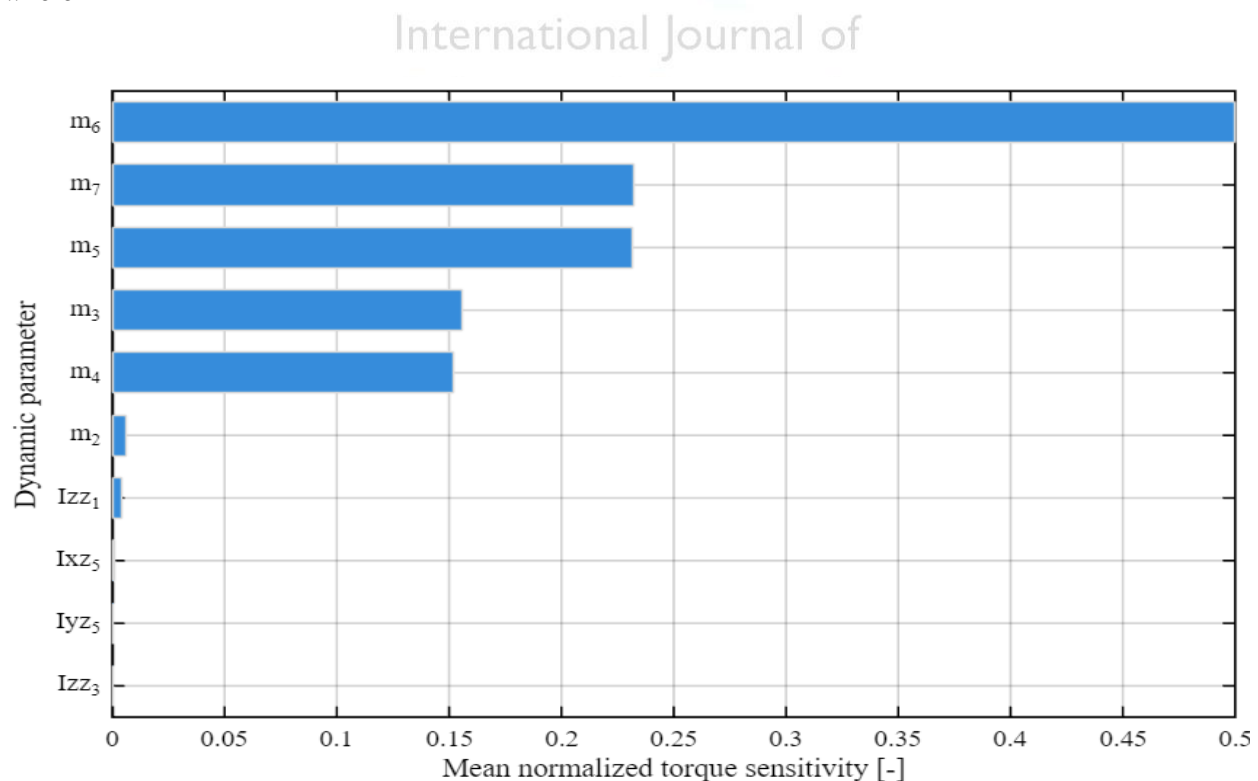


Fig. 1. Parameter-wise dynamic sensitivity ranking of the 49 dynamic parameters.

The five dominant masses account for 98.672% of the total mean sensitivity, while the ten most influential parameters account for 99.589%. This concentration is the primary justification for the sensitivity-weighted DUSI. Rather than treating all 49 parameters equally, the proposed metric emphasizes the parameters that have been demonstrated to dominate the torque response.

The largest sensitivity contribution is associated with m_6 , which receives a weight of 0.39327. The next largest contributions are associated with m_7 and m_5 . The remaining parameters have considerably smaller contributions.

This result also demonstrates why a parameter-wise analysis is necessary before robustness optimization. An equal-weight uncertainty metric would assign the same importance to parameters that have very different effects on the torque response. The proposed weighting instead reflects the dynamic behavior of the investigated trajectory.

4.2 Null-Space Sensitivity

The calculated null-space direction satisfied the condition $Jn = 0$ with a maximum residual of

5.30×10^{-16} , confirming numerical consistency of the redundancy direction.

The local DUSI gradient was negative at 45 of the 51 evaluated samples, corresponding to 88.2% of the trajectory. The mean gradient was $-0.021609 \text{ rad}^{-1}$, indicating that positive redundant displacement generally moves the manipulator toward lower dynamic sensitivity.

The finite-difference convergence study produced virtually identical mean gradients for perturbations of 10^{-4} , 10^{-3} , and 10^{-2} rad . The relative difference remained below 0.004%. This confirms that the identified sensitivity direction is numerically stable.

The result is significant from a design perspective. The null-space motion is not merely an additional kinematic degree of freedom; it provides a mechanism for changing the dynamic sensitivity of the robot without intentionally changing the primary Cartesian trajectory.

4.3 Optimized Redundant Trajectory

The optimized B-spline control points were $c =$

$[0, 0.0172, 0.0500, 0.0500, 0.0266, 0.0034, 0.0012, 0] \text{ r}$
(37)

The resulting redundancy trajectory is shown in Fig. 2.

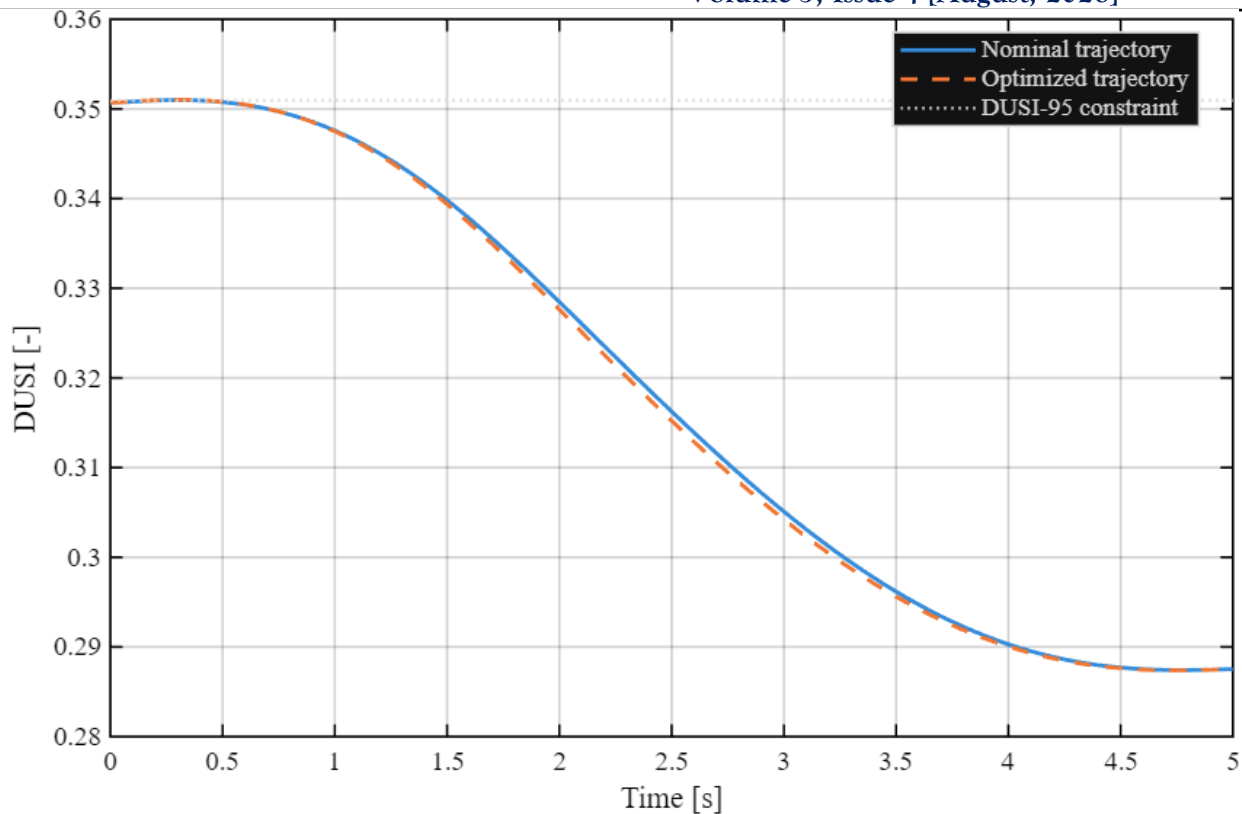


Fig. 2. Optimized null-space displacement over the 5 s trajectory.

The optimized displacement remains within the ± 0.05 rad constraint and varies smoothly throughout the trajectory. The zero initial and final control points ensure that the redundant

correction returns to zero at the trajectory boundaries.

The quantitative comparison between the nominal and optimized trajectories is presented in Table I.

Table I. Performance comparison of nominal and optimized trajectories

Metric	Nominal	Optimized	Change
Mean DUSI	0.318140	0.317730	-0.1288%
DUSI-95	0.350915	0.350939	+0.0070%
Maximum DUSI	0.350995	0.351025	+0.0085%
Mechanical work (J)	0.466584	0.482265	+3.361%
RMS jerk	0.179935	0.173920	-3.343%
Peak torque norm (N·m)	13.2816	13.2865	+0.0367%
Maximum position error (mm)	0	0.0917	—
Maximum pose error (rad)	~ 0	1.439×10^{-4}	—

The optimized trajectory reduces mean DUSI by 0.1288%. This confirms that the optimizer successfully exploits the redundant configuration to move toward a lower average sensitivity region.

The DUSI-95 value increases slightly by 0.0070%, but remains below the imposed upper-tail limit. The maximum DUSI increases by only 0.0085%. Thus, the reduction in average sensitivity is not accompanied by a substantial deterioration of the

upper-tail response. Figure 3 shows the corresponding torque-norm profiles.

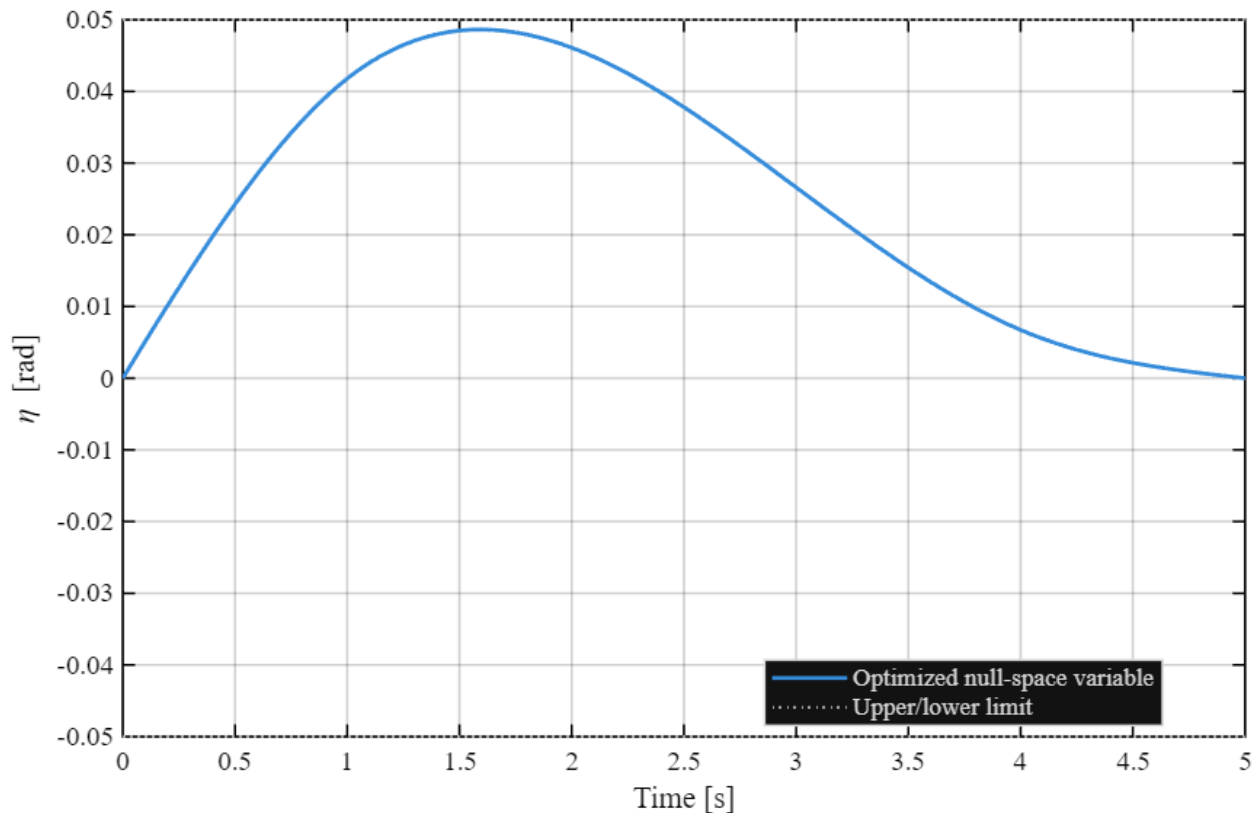


Fig. 3. Nominal and optimized torque-norm profiles.

The two torque profiles remain closely matched. The peak torque norm changes by only 0.0367%, indicating that the optimized redundant motion does not introduce a significant increase in peak dynamic demand.

The principal cost of the optimization is mechanical work. Absolute mechanical work increases by 3.361%. In contrast, RMS jerk decreases by 3.343%. Therefore, the optimized trajectory represents a robustness-oriented compromise rather than a simultaneous improvement of all performance measures.

The maximum Cartesian position error is only 0.0917 mm, substantially below the 1 mm

constraint. This confirms that the redundant modification preserves the intended Cartesian trajectory.

4.4 Monte Carlo Robustness Validation

The deterministic results establish that the optimized trajectory reduces mean DUSI, but they do not by themselves demonstrate improved performance under actual parameter perturbations. An independent Monte Carlo analysis was therefore performed using 1000 paired uncertainty realizations.

The resulting RMS relative torque deviations are presented in Fig. 4.

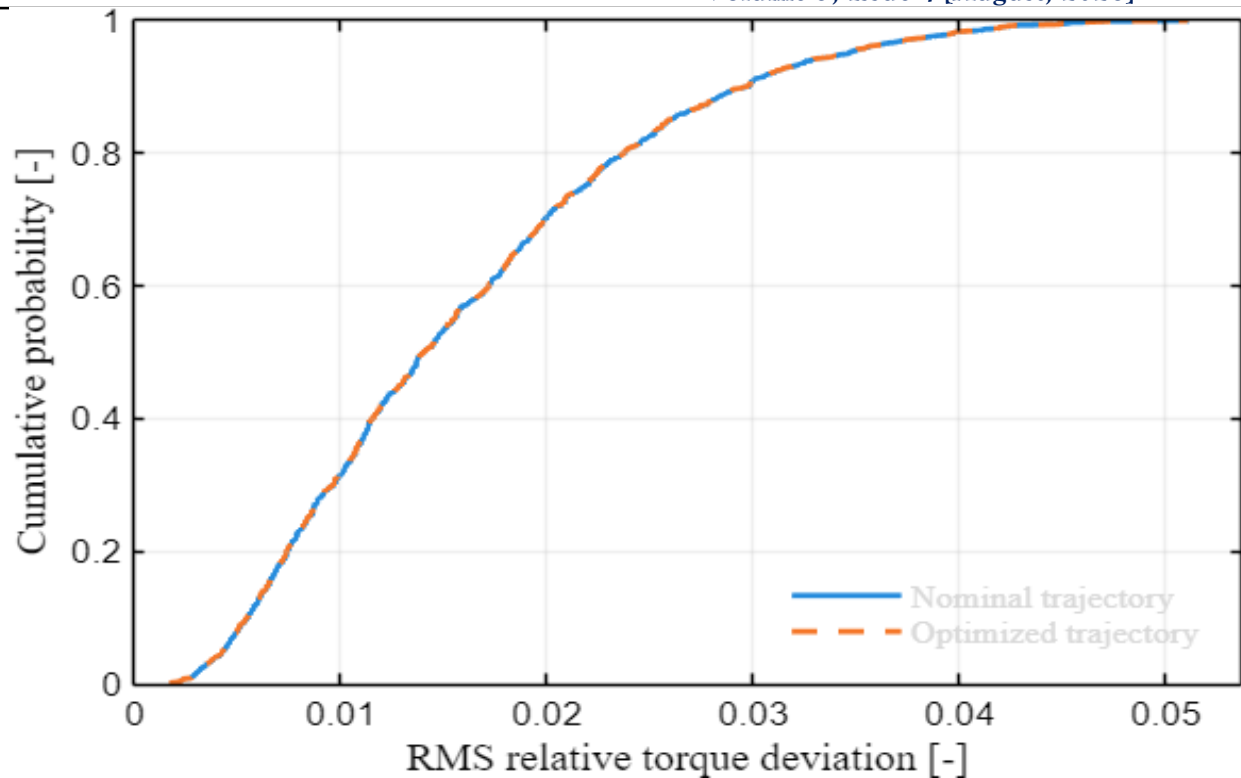


Fig. 4. Distribution of RMS relative torque deviation under $\pm 5\%$ dynamic-parameter uncertainty.

The mean RMS relative torque deviation decreases from 0.01603272 for the nominal trajectory to 0.01600113 for the optimized trajectory, corresponding to a 0.197% reduction.

The 95th-percentile RMS deviation decreases by 0.0714%. The paired comparison is shown in Fig. 5.

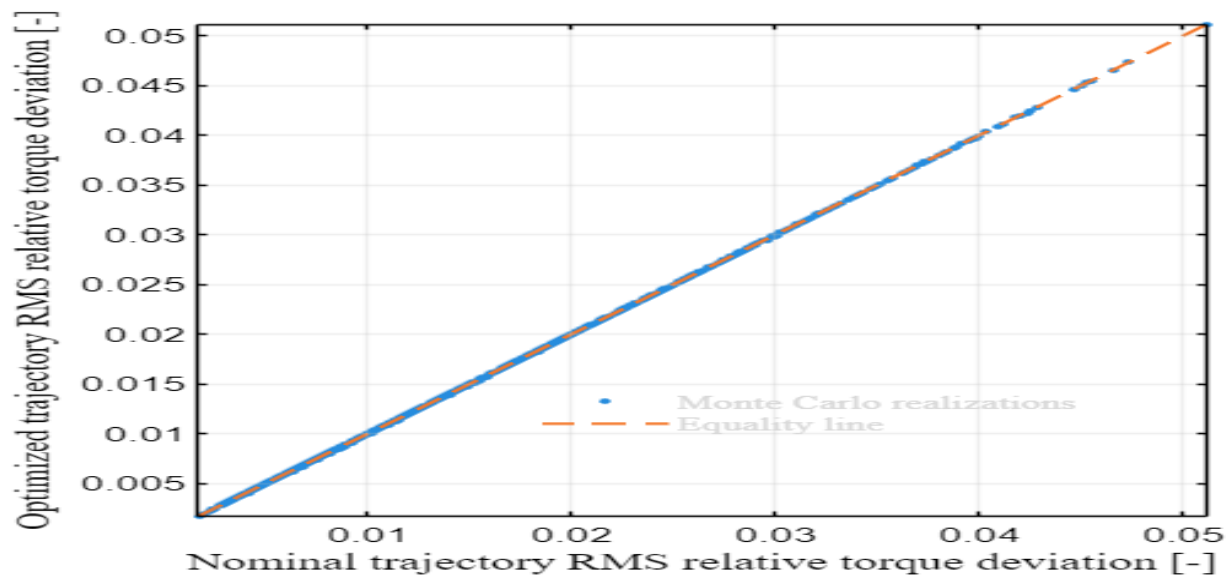


Fig. 5. Paired comparison of RMS relative torque deviation for the nominal and optimized trajectories.

The optimized trajectory produces a lower RMS uncertainty response in 917 of the 1000 paired realizations, corresponding to a 91.7% win rate.

The principal Monte Carlo results are summarized in Table II.

Table II. Monte Carlo validation under $\pm 5\%$ dynamic-parameter uncertainty

Metric	Nominal	Optimized
Mean RMS relative torque deviation	0.016033	0.016001
P95 RMS relative torque deviation	0.034650	0.034625
Maximum RMS relative torque deviation	0.051136	0.051106
P95 peak torque (N·m)	13.6531	13.6588
P95 mechanical work (J)	0.4754	0.4917
Mean maximum absolute torque deviation (N·m)	0.21668	0.21680

The paired t -test gives $p = 5.84 \times 10^{-175}$, while the Wilcoxon signed-rank test gives $p = 3.41 \times 10^{-149}$. The paired Cohen's d_z is 1.103, and the 95% confidence interval of the mean paired difference is approximately $[2.981, 3.337] \times 10^{-5}$.

These results indicate that the improvement is statistically consistent across the paired uncertainty realizations. However, its engineering magnitude is modest. The mean RMS uncertainty reduction is only 0.197%; therefore, statistical significance should not be interpreted as evidence of a large robustness improvement.

The Monte Carlo results also reproduce the deterministic trade-off. The 95th-percentile peak torque increases by only 0.0420%, whereas the 95th-percentile mechanical work increases by 3.440%. Thus, the robustness improvement is obtained primarily through a change in redundant configuration rather than a reduction in the overall mechanical demand.

Importantly, the Monte Carlo samples were not used during optimization. The stochastic experiment therefore provides an independent validation of the deterministic sensitivity-guided design rather than simply confirming that the optimizer minimized the same samples used to evaluate it. The resulting small but consistent reduction in uncertainty-induced torque variation supports the proposed sensitivity-to-redundancy design mechanism. The validated Monte Carlo findings are consistent with the final frozen results in the submitted draft.

4.5 Optimization Reproducibility

The final optimization was repeated from five different initial control-point configurations. All five runs converged to essentially the same solution. The objective-function range was only 3.60×10^{-12} , while the mean-DUSI range was 3.20×10^{-9} . This consistency demonstrates strong numerical reproducibility within the tested initialization set. It also supports the stability of the obtained solution for the selected B-spline representation and constraint set.

The result should not, however, be interpreted as proof of global optimality. Multiple-start convergence demonstrates repeatability of the identified solution but cannot exclude the existence of a better solution outside the tested initializations or parameterization. The five-start result and the frozen Phase 5E solution are documented in the validated draft.

4.6 Discussion

The results establish a consistent relationship between dynamic-parameter sensitivity and redundant motion. The parameter-wise analysis first identifies where the torque response is most sensitive. The five dominant masses account for nearly the entire mean sensitivity contribution, allowing the DUSI to focus on the most influential uncertainty directions.

The null-space analysis then demonstrates that the redundant degree of freedom can modify this sensitivity. The predominantly negative DUSI gradient indicates that suitable redundant motion

can reduce dynamic sensitivity without requiring a change in the primary Cartesian objective.

The optimized trajectory confirms that this local sensitivity information can be converted into a feasible global redundant motion. The mean DUSI reduction is small, but it is achieved while satisfying the upper-tail DUSI constraint, Cartesian accuracy requirement, redundancy bound, joint constraints, and mechanical-work limit.

The Monte Carlo experiment provides independent evidence that the deterministic DUSI reduction has a corresponding effect under uncertain dynamic parameters. The improvement is small but occurs in 91.7% of the paired uncertainty realizations. This is an important distinction: the contribution of the method is not a claim of dramatic robustness improvement, but a systematic mechanism for obtaining a measurable robustness benefit from an otherwise unused redundant degree of freedom.

The results also reveal an explicit robustness-work trade-off. The optimized trajectory requires approximately 3.36% more nominal mechanical work, while reducing mean DUSI and RMS jerk. This trade-off should be considered when selecting the objective weights for applications in which energy consumption is more important than robustness.

Overall, the study demonstrates that redundancy can be treated as a dynamic design variable rather than solely as a kinematic resource. The sensitivity ranking provides the uncertainty information, the null-space gradient provides the direction of beneficial configuration change, and the B-spline optimization provides a smooth and constrained implementation.

5. Conclusion

This paper presented a dynamic-sensitivity-guided trajectory optimization framework for a redundant 7-DOF manipulator under dynamic-parameter uncertainty. The proposed approach integrates parameter-wise sensitivity analysis, sensitivity-weighted DUSI, null-space sensitivity analysis, B-spline redundancy parameterization, constrained

nonlinear optimization, and independent Monte Carlo validation.

The 49-parameter sensitivity analysis showed that the dynamic response of the investigated trajectory is strongly concentrated in five link masses. These five parameters accounted for 98.672% of the total mean sensitivity, providing a quantitative basis for the sensitivity weighting used in DUSI.

The null-space analysis demonstrated that redundancy can be used to modify dynamic sensitivity. A negative local DUSI gradient was obtained at most trajectory samples, indicating that an appropriate redundant displacement can move the manipulator toward configurations with lower dynamic sensitivity while preserving the primary task.

The final B-spline optimization reduced the mean DUSI by 0.1288%. The DUSI-95 value increased by only 0.0070% and remained within the prescribed upper-tail constraint. The maximum Cartesian position deviation was 0.0917 mm, while the peak torque norm changed by only 0.0367%. The optimized trajectory therefore achieved the intended sensitivity reduction without a substantial change in peak torque or Cartesian accuracy.

The optimization produced a moderate increase in mechanical work of 3.361%, while RMS jerk decreased by 3.343%. This demonstrates that the proposed approach provides a controlled robustness-work trade-off rather than improving all trajectory measures simultaneously.

Independent Monte Carlo validation using 1000 paired $\pm 5\%$ uncertainty realizations showed a 0.197% reduction in mean RMS relative torque deviation and a 0.0714% reduction in its 95th percentile. The optimized trajectory outperformed the nominal trajectory in 91.7% of the paired realizations. The statistical tests confirmed that the difference was highly consistent, although the engineering magnitude of the robustness improvement remains modest.

The principal contribution of this work is therefore the methodology that connects dynamic-parameter sensitivity with redundant trajectory design. The approach provides a systematic

procedure for identifying influential dynamic parameters, determining how redundancy affects their sensitivity, and incorporating that information into a constrained trajectory optimization problem.

The present study is limited to simulation of the Franka Panda and does not include experimental validation. In addition, the independent $\pm 5\%$ perturbation of inertia components does not explicitly guarantee physically realizable positive-definite inertia tensors. The DUSI ranking is also dependent on the selected trajectory and operating conditions. Future work should therefore investigate physically consistent uncertainty sets, experimental validation, varying payload conditions, and task-dependent sensitivity weighting.

The results demonstrate that the redundant degree of freedom of a 7-DOF manipulator can be exploited for dynamic-robustness-oriented trajectory design. The proposed framework achieves a small but consistent reduction in dynamic sensitivity and uncertainty-induced torque variation while maintaining explicit trajectory feasibility constraints. This provides a practical basis for extending redundancy resolution from purely kinematic optimization toward uncertainty-aware dynamic trajectory planning.

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