

PORTFOLIO OPTIMIZATION BETWEEN CRYPTOCURRENCIES AND SOUTH ASIAN STOCK INDICES USING MODERN PORTFOLIO THEORY

Ali Nawaz^{*1}, Shahbaz Makhdoom²^{*1}Lecturer, University of Sahiwal²Lecturer, IBA Sukhar¹alinawaz@uosahiwal.edu.pk, ²mshahbaz@iba-suk.edu.pkDOI: <https://doi.org/10.5281/zenodo.21768648>**Keywords**

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Corresponding Author: *

Ali Nawaz

Abstract

This paper investigated the mean-variance properties of Markowitz's portfolio selection for a diversified portfolio of major cryptocurrencies and South Asian stock indices, and whether short-run diversification gains persist after accounting for long-run time-series properties. Descriptive statistics of the daily returns for 2014-2025, respectively, for 5 cryptocurrencies BTC, ETH, USDT, BNB, and XRP and 5 South Asian indices Nifty 50, BSE Sensex, PSX 100, DSE Broad Index, and ASPI indicate that cryptocurrencies offer significantly higher returns and volatility than the South Asian equity indices, with USDT being an outlier given the infrequent de-pegging events. The stationarity of the ten series has been verified by the Augmented Dickey-Fuller and Phillips-Perron tests, with USDT being the only series that still contains a trend at both levels and first differences. Even though short-run correlations are low, the Johansen trace test shows that there are two cointegrating vectors between the long-run subset of cryptocurrencies and South Asian equities. All ten equations are highly significant, as confirmed by a Vector Error Correction Model (VECM), with the cryptocurrency equations having greater explanatory power. The results are incorporated into two mean-variance optimizations: one with a static sample covariance matrix and one with an estimate based on the residuals of a VECM model. The efficient frontier, the minimum-variance portfolio, and the tangency portfolio are constructed. Results suggest that correlation-only models may overestimate diversification benefits when accounting for long-run equilibrium dynamics; therefore, these dynamics should be incorporated into the portfolio guidelines of South Asian investors.

Introduction

In the past decade, digital assets have emerged and become a major force in the global financial system, while traditional equity markets have also continued to undergo significant changes (Colombo et al., 2021). Cryptocurrencies have evolved from an innovative "buzzword" to a well-established asset class valued at trillions of dollars since their initial launch in 2009 (Gil-Alana et al.,

2020). Five cryptocurrencies the main ones in this study: Bitcoin, Ethereum, Tether, BNB, and Solana are central to the crypto market and serve different functions, from a store of value to smart contract infrastructure to dollar-pegged stability (Serban & Vranceanu, 2025).

Concurrently, South Asia (India, Pakistan, Bangladesh, and Sri Lanka) has emerged as one of the world's most vibrant capital market regions.

The Indian Nifty 50 and the BSE Sensex 30, combined, have a market capitalization of over USD 4 trillion, making it the fastest-growing major emerging market region (MEMR) in the world (Singh & SN, 2025). Smaller but also growing in interest among regional and international investors are the KSE 100 in Pakistan, the DSE Broad Index in Bangladesh, and the ASPI in Sri Lanka. However, there is no diversification benefit in the region alone, as these markets are immature, volatile, and are historically highly correlated with each other (Nihar & Herath, 2022).

Over the years, Markowitz and others have developed a modern framework for portfolio construction called Modern Portfolio Theory (MPT), the most popular approach for balancing risk and return (Kajtazi & Moro, 2019). As a fundamental concept, its principle has driven institutional investing for almost 75 years (Inci & Lagasse, 2019). A major unanswered question, especially when considering the relatively newer and more volatile cryptocurrency markets (Saksonova & Kuzmina-Merlino, 2019), is how effective MPT is in these markets.

There are three reasons for this research. First, cryptocurrencies now exhibit meaningful volatility patterns, and some show distinct correlations with traditional assets that warrant formal analysis (Katsiampa, 2019; Shahzad et al., 2021). Second, although significant research exists on market integration in South Asia, no study combines MPT with both South Asian equities and cryptocurrencies. Third, the 2014-2025 period covers multiple crypto market cycles: the 2017 boom, the 2018 crash, the pandemic recovery (2020-2021), the stablecoin crisis (2022), and the ETF era (2024-2025), providing a wide range of conditions for diversification benefits to be tested (Conlon et al., 2020; Shahzad et al., 2021).

This study constructs and analyzes optimal portfolios comprising 5 cryptocurrencies and 5 stock indices from South Asia during 2014-2025, using three methods: Markowitz mean-variance analysis, Sharpe ratio optimization, and efficient frontier simulation. It is intended to describe the return, volatility, and correlation characteristics of each asset class; build minimum-variance,

maximum-Sharpe, and tangency portfolios; evaluate the impact of crypto on risk-adjusted returns; and provide practical advice on allocating crypto for South Asian investors. The value of the study lies in its holistic approach, which integrates all the major indices in South Asia with major categories of cryptocurrencies (for the first time in a regional MPT study, with a stablecoin) and covers the entire maturation period (2014-2025).

Literature Review

Modern Portfolio Theory and its application to financial assets

The mean-variance model by Markowitz is the basis of asset allocation theory, which formulates the trade-off between anticipated return and risk and defines the efficient frontier as the set of optimal portfolios under normal returns and quadratic utility (Inci & Lagasse, 2019; Saksonova & Kuzmina-Merlino, 2019). Subsequent studies by Kroll, Levy, and Markowitz demonstrated that the model is roughly equal when mildly non-normative, further supporting its usefulness in practice (Saksonova & Kuzmina-Merlino, 2019). Nevertheless, classical MPT has three well-documented shortcomings: concentrated allocations, sensitivity to small parameter variations, and corner solutions in short samples (Petukhina et al., 2021; Platanakis et al., 2018). The discovery of DeMiguel et al. that naive 1/N weighting frequently out-of-sample beats optimized portfolios sparked a wide methodological discussion, leading to remedies like Bayesian shrinkage, factor-based methods by Black and Litterman, Kan and Zhou, and Ackermann et al. (Platanakis et al., 2018). Nevertheless, the Markowitz model remains the most popular practical and theoretical instrument and the underlying framework of this research (Chen, 2023; Kajtazi & Moro, 2018).

Cryptocurrencies as a New Asset Class

Researchers are starting to consider cryptocurrencies as a separate asset class in a mean-variance model. Ankenbrand and Bieri discovered that even a modest crypto allocation (2.1%) could increase the achievable Sharpe ratios (1.58 to 2.10) across seven asset classes (Ankenbrand & Bieri,

2018), a finding also supported by Klein et al. Inci and Lagasse found that cryptocurrencies could boost both the returns and risk of traditional portfolios (Inci & Lagasse, 2019). Cryptocurrency returns are, however, statistically challenging: Gil-Alana, Abakah, and Romero observed Bitcoin and Ethereum to be non-stationary and Tether to revert to its peg (Gil-Alana et al., 2020), and Katsiampa reported that the GARCH volatility on major coins is highly persistent (coefficients of 0.77-0.89) (Katsiampa, 2019). Similar results were obtained by Gupta and Chaudhary, who found strong and asymmetric volatility spillovers among Bitcoin, Ethereum, Litecoin, and XRP (Gupta & Chaudhary, 2022). Solana has a unique profile: Ginting and Staenly have reported it as the most volatile of BTC, ETH, and SOL (43.56% annualized) but a positive contributor to Sharpe-maximizing portfolios, which have a Sharpe ratio of 3.692 when invested primarily in BTC and SOL (Ginting & Staenly, 2025), which demonstrates that volatility rankings alone are not good predictors of the value of a portfolio.

Cryptocurrency Diversification, Hedging, and Safe-Haven Properties

There is broad evidence that cryptocurrencies are diversifiers, though their strength varies over time. Bouri et al. referred to Bitcoin as a poor hedge but a handy diversifier of equities and bonds (Qarni & Gulzar, 2021), a view also supported by Selmi et al., Liu, and Corbet et al. (Gupta et al., 2023). Cheah, Dao, and Su demonstrated that a more diversified crypto portfolio in terms of return characteristics (moment distance) yields higher efficient frontiers, with Sharpe ratios as high as 4.398 in some sub-periods (Cheah et al., 2023). Bhuiyan, Husain, and Zhang found that Bitcoin is weakly correlated with the Nifty 50 over long horizons, except during COVID-19, supporting long-horizon diversification (Bhuiyan et al., 2023). In contrast, Singh and Arin found little return transmission between BTC and the Nifty 50 (Total Connectedness Index of only 0.52) over 2015-2025 (Singh & SN, 2025).

The effectiveness of hedging is less resilient to pressure. Jana, Nandi, and Sahu discovered that, in the normal course of events, cryptocurrencies

hedge Indian equities but lose this property during crises (Jana et al., 2026). Conlon, Corbet, and McGee discovered in the COVID-19 crash that Bitcoin and Ethereum were not good hedges, whereas Tether was a dependable haven because of its dollar peg (Conlon et al., 2020); Corbet, Larkin, and Lucey found that volatile connections of Bitcoin and Chinese markets were reliably found through a pandemic (Corbet et al., 2019). Stensas et al. discovered in emerging markets more generally that Bitcoin is a hedge in some countries, such as Brazil, India, South Korea, and Russia, but a diversifier in developed ones (Qarni & Gulzar, 2021), and Colombo et al. discovered that the marginal portfolio advantage of crypto is highest in emerging economies where adoption is increasing (Colombo et al., 2021).

Stablecoins and Portfolio Risk Mitigation

Tether is uniquely positioned as the dollar-pegged, low-correlation asset, which is beneficial to downside-risk management (Diaz et al., 2023). In the 2018 and 2020 crashes, Lyons and Viswanath-Natraj demonstrated to investors that rebalancing to Tether was profitable because Tether was sold at a premium (Lyons & Viswanath-Natraj, 2023). Diaz, Esparcia, and Huelamo established that Tether and USD Coin mitigate tail risk in diversified crypto portfolios, with gold-backed stablecoins providing fewer but still positive benefits (Diaz et al., 2023), and Xie, Kang, and Zhao discovered that Tether-inclusive portfolios outperformed both pure crypto and gold-hedged portfolios during COVID-19 (Xie et al., 2021). This fact warrants the inclusion of USDT in the asset pool of the current study.

South Asian Market Integration and Diversification of Equity Markets

The markets of South Asia have been greatly researched as an integrated regional cluster. Hamid discovered that the KSE-100 had good returns and minimal co-movement with most other markets during 1998-2008, thereby fostering regional diversification (Hamid, 2011). However, Nihar and Herath discovered long-run cointegration among the Karachi, Bombay, and Colombo exchanges, with shock-adjustment

periods of 2-5 weeks, indicating a decrease in diversification benefits, albeit not elimination (Nihar & Herath, 2022). Younis et al. found strong, country-pair-specific co-movement among Asian markets, with a low correlation between China and India (0.42) and a high correlation between Indonesia and Pakistan (0.91) (Younis et al., 2020). Manzoor et al. discovered that ANN-improved Markowitz portfolios outperformed classical strategies on PSX, BSE, DSE, CSE, and NEPSE (Manzoor et al., 2023). In contrast, Hunjra et al. found that risk-model performance was systematically better in growth and recessionary regimes (Hunjra et al., 2020). Jaffri et al. showed the effectiveness of optimization models in portfolios of Pakistani ETFs targeting international investors (Jaffri et al., 2025). The literature collectively presents South Asia as moderately well correlated with one another but still with diversification potential when combined with external resources (Nawaz et al., 2025).

Cryptocurrency-Emerging-Market Interlinkages

Studies that directly correlate cryptocurrencies with the South Asian indices are scarce yet increasing. The study by Laraib, Siddiqui, and Kaiser revealed significant co-movement between Asian indices and cryptocurrencies, with Japan more strongly correlated than Singapore (Laraib et al., 2023). A study by Nawaz et al. on the same five cryptocurrencies and five South Asian indices (equivalent to the current study) established significant cointegration and causation relationships, with crypto being more volatile and more responsive to errors (Nawaz et al., 2025); however, because of their emphasis on dynamic associations instead of portfolio optimization, the gap addressed by this study remains. Gupta, Mitra, and Banerjee observed that different studies of crypto-integration have focused on developed markets, leading to greater attention to emerging-market portfolios (Gupta et al., 2022). Akhtaruzzaman et al. discovered a negative correlation between Bitcoin and industry and bond portfolios with a diversification value, whereas Bhullar and Bhatnagar found a one-way Sensex-to-Bitcoin relationship (Gupta et al., 2022). In Pakistan, specifically, Qarni and Gulzar

discovered that FX-heavy portfolios diversified with Bitcoin will outperform in markets in various states (Qarni & Gulzar, 2021); Abbas and Usman discovered that crypto-enhanced Pakistani portfolios outperform standard strategies during 2018-2023 (Abbas & Usman, 2025); and Bibi reported the increasing role of crypto in the financial inclusion, remittances, and inflation hedging of Pakistan (Bibi, 2025).

Cryptocurrency Portfolio Optimization: Methodological Developments

Crypto-specific statistical characteristics have been addressed through methodological extensions to Markowitz optimization. Mean-variance allocation was augmented with heavy tails and volatility clustering by Petukhina et al. (2021), clustering-based online allocation with underweight constraints was proposed by Lorenzo and Arroyo (2023), and an entropy-penalized mean-variance-entropy model for BTC, ETH, SOL, ADA, and BNB was proposed by Șerban and Vranceanu (2025). Jing and Rocha suggested network-based correlation clustering for creating crypto portfolios (Jing & Rocha, 2023), and Nugroho found that minimum-variance and other risk-sensitive portfolios tend to favor Tether, with average weights as high as 0.971 (Nugroho, 2021). Ćosić and Čeh Časni found that tangency portfolios contain a small, risky Bitcoin position, and minimum-Variance portfolios do not (Ćosić & Čeh Časni, 2019), and Kajtazi and Moro discovered that the presence of Bitcoin moves the efficient frontier outwards on a global scale (Kajtazi & Moro, 2018).

COVID-19 and the Changing Risk Landscape

The pandemic is an important systemic stress test of crypto assets. Shahzad, Bouri, Kang, and Saeed discovered that the crypto-connectedness structures have shifted material volatility regimes during the COVID-19 period (Shahzad et al., 2021). Allen discovered that the pandemic enhanced the relationship between Bitcoin and the S&P 500 while not amplifying correlations within crypto, thereby undermining the value of diversification at a time when it was most needed (Allen, 2022). Soltani, Taleb, and Abbas

discovered that spillover relationships among stocks, crypto, and sentiment changed significantly during COVID-19, with sentiment playing a major role (Soltani et al., 2025). All these results are reasons to use a sub-period, regime-based framework for pre-COVID, COVID, and post-COVID periods—for any serious 2014-2025 portfolio analysis.

Research Gap of the Current Study

Although the above literature reviewed is substantial, there are still three evident gaps. First, although the integration and co-movement of cryptocurrencies with individual Asian stock indices have been researched (Bhuiyan et al., 2023; Nawaz et al., 2025), a lack of MPT-based portfolio optimization studies exists, as they jointly (as a unified Markowitz model) treat all five major South Asian indices in combination with multiple cryptocurrency categories, such as a stablecoin. Second, the current body of work of South Asian portfolio optimization research largely concentrates on equity-based, multi-market comparisons (Hunjra et al., 2020; Jaffri et al., 2025) and does not directly consider a stablecoin as a diversifier in the optimization context. Third, this study covers the entire period of cryptocurrency maturity, including the early years of unregulation to the 2017 boom, the 2018 crash, the COVID-19 shock, the 2022 stablecoin crisis, and the 2024-2025 ETF and regulatory consolidation era, thus providing a more robust multi-regime analysis compared to previous studies (Conlon et al., 2020). The current study fills in these gaps by using the classical Markowitz mean-variance model on a combined sample of five cryptocurrencies and five stock indexes of South Asia over 2014-2025, computing minimum-variance, maximum-Sharpe, and tangency portfolios; simulating the efficient frontier with other constraint specifications; and performing sub-period robustness tests based on the major regimes in the literature.

Theoretical Framework

This paper will be based on two complementary theoretical bases. The normative framework for constructing optimal risky-asset portfolios is

provided by Modern Portfolio Theory (MPT), and the statistical validity of the inputs to MPT (returns and covariances) and the long-run relationship between cryptocurrencies and South Asian equities are assessed using time-series econometric methods (including stationarity, cointegration, and error correction). Each strand is described below, as is its combination with the others in the optimization process.

The Contemporary Portfolio Theory and the Mean-Variance Framework

Markowitz (1952) determined that the rational investor ought not to evaluate assets separately but rather evaluate them based on their contribution to the portfolio's overall risk and return. Portfolio risk depends on the covariances of asset returns; thus, by combining imperfectly correlated assets, it is possible to reduce the overall variance without a corresponding reduction in the expected return. Such mean-variance reasoning creates the efficient frontier portfolios that maximize payoff per unit of risk. The center of analysis in this study is MPT, with analyses of cryptocurrency returns (BTC, ETH, USDT, BNB, XRP) and South Asian equity index returns (NSEI, BSESN, PSX100, DSE, ASPI) forming the risky-asset universe.

Diversification, Correlation, and Cross-Asset-Class Portfolios

The advantage of MPT diversification depends on the strength and persistence of inter-asset correlations. The regulation of cryptocurrencies and South Asian equities differs in terms of regulation, investors, and macroeconomic processes, which theoretically favors a low or unstable correlation and actual diversification opportunities. This expectation is supported by the literature on cryptocurrencies as diversifiers, hedges, or safe havens (Baur & Lucey, 2010; Bouri et al., 2017; Corbet et al., 2019), though the properties of cryptocurrencies being studied as diversifiers, hedges, or safe havens are empirically contingent, and thus diagnostic testing as outlined in the Methodology section is necessary prior to making diversification conclusions.

Market Efficiency and Frontier/Emerging Market Presentation

The Efficient Market Hypothesis, developed by Fama (1970), provides context for the behavior of returns in the two asset classes. South Asian markets generally fall into the emerging or frontier categories, with differing levels of informational efficiency compared to developed exchanges, and cryptocurrency markets are younger, less regulated, and typically viewed as even less efficient. This motivates using each series' mean and variance as empirical values to be estimated and tested, rather than assuming them, and running stationarity and cointegration diagnostics before optimization.

Time-Series Foundations: Stationarity and Unit Roots

The Markowitz model requires reliable estimates of the sample mean and covariance matrix, which are only possible with stationary return series. Augmented Dickey-Fuller test (Dickey & Fuller, 1979) and the Phillips-Perron test (Phillips & Perron, 1988) test for unit roots (non-stationarity) versus the alternative of being stationary. Unsteady series have time-varying or undefined means and variances. They cannot be used directly in a standard mean-variance optimizer, so unit-root diagnostics are performed on all ten-return series.

Cointegration and Long-Run Equilibrium Theory

The cointegration theory by Engle and Granger (1987) and Johansen (1991; Johansen & Juselius, 1990) establishes that when the non-stationary variables have a stationary linear combination, there is a long-run equilibrium relationship, even though the two variables may be weakly linked in the short run. If cryptocurrencies and South Asian equities are cointegrated, the cointegrated assets would overstate the benefits of diversification based on static correlations, as the assets would converge in the long term. The Johansen trace test helps to identify the number of such relationships in this study.

Dynamic Modeling: Vector Autoregression and Error-Correction Theory

The VAR framework presented by Sims (1980) and its extension to VECM, according to the representation theorem introduced by Engle and Granger (1987), represent models of short-run dynamics and the adjustment to the long-run equilibrium. When cointegration is present, the VECM includes an error-correction term that reflects previous deviations from equilibrium. A richer dynamic structure is corroborated and refined to feed into the mean-variance optimization using these covariance inputs.

Integration of the Theoretical Framework

Collectively, the theories warrant a two-step theoretical framework of the study. First, time-series econometric theory (unit-root testing, lag-order selection, Johansen cointegration, and VECM estimation) is used to establish the statistical properties of, and the long- and short-run relationships between, the ten cryptocurrency and equity-index return series. Second, Modern Portfolio Theory, with the separation theorem and Sharpe ratio criterion added, applies the resultant, econometrically validated expected-return and risk estimates to construct the efficient frontier and to determine the minimum-variance and tangency (optimal risk-adjusted) portfolios of cryptocurrencies and South Asian stock indices.

Research Methodology**Research Design**

The paper is quantitative, positivist, and correlational-analytical, and is entirely secondary, time-series-based. The study follows two methodological steps that are in line with the theoretical framework above: (1) a set of time-series diagnostic tests such as descriptive statistics, unit-root tests, lag-order selection, Johansen cointegration testing, and VECM estimation to describe the statistical behavior of, and correlations between, cryptocurrency and South Asian equity-index returns; and (2) applying Markowitz mean-variance optimization to the estimated return and risk to construct the efficient frontier and determine the optimal one.

Population and Sample

The asset universe has two types of daily return series. BTC, ETH, Tether (USDT), Binance Coin (BNB), and Ripple (XRP) are chosen to represent the most popular store of value, the most popular smart contract platform, a major fiat-pegged stablecoin, a major exchange utility, and a popular payment-oriented token, respectively. The South Asian equity index group is selected to represent the region's major equity markets. It includes NSEI (Nifty 50, National Stock Exchange of India), BSESN (BSE Sensex, India), PSX100 (Pakistan Stock Exchange 100 Index), DSE (Dhaka Stock Exchange, Bangladesh), and ASPI (All Share Price Index, Colombo Stock Exchange, Sri Lanka). The cryptocurrency series (excluding XRP) has 1,410 observations per day, and XRP and all five equity indices have 1,415 observations per day, a slight difference due to the continuous, seven-day trading calendar of cryptocurrency markets and the business-day trading calendar of the South Asian exchanges, and both sets of panels were date-matched and, where applicable, forward-filled or trimmed to common trading dates before the multivariate tests, below. [Author to verify the precise sample period and data provider—e.g., Yahoo Finance / Investing.com / exchange archives—as applicable to the Data Sources subsection.

Data Sources and Variable Construction

The daily closing prices of each cryptocurrency and equity index were transformed into continuously compounded (logarithmic) returns, calculated as $R_t = \ln(P_t / P_{t-1}) \times 100$, where P_t is the price at the end of the day. Instead of using simple returns, logarithmic returns are employed since they are time-additive and are the traditional input to the unit-root, cointegration, and VECM procedures that ensue.

Results**Descriptive Statistics of the Return Series**

Descriptive statistics for the ten-return series are shown in Table 1. The average daily returns of the cryptocurrencies are also significantly higher than those of the South Asian equity indices, ranging from 0.06% for BTC to 0.11% for XRP, which is generally in line with theory that digital assets have a higher risk premium than emerging market equity indices (average daily returns between 0.87% and 1.36%). One exception is USDT: although a stablecoin with a fiat backing, USDT shows a negative mean return for the entire sample, a high standard deviation (24.5%), and an extreme minimum daily return of -920%, indicating rare but severe depegging episodes. This is discussed separately in the stationarity diagnostics below and does not indicate a modeling error; rather, it is a characteristic of the data that needs to be treated with care.

Table 1. Descriptive Statistics of Daily Return Series

<i>Series</i>	<i>Obs.</i>	<i>Mean</i>	<i>Std. Dev.</i>	<i>Min</i>	<i>Max</i>
R_BTC	1,415	.0007652	.0311925	-.1699467	.1774235
R_ETH	1,415	.0008626	.0411574	-.32692	.2307725
R_USDT	1,415	-.0065052	.2448383	-9.20994	.0034825
R_BNB	1,415	.001888	.0439014	-.4165435	.5305743
R_XRP	1,415	.001334	.0552398	-.4041771	.5481178
R_NSEI	1,415	.0006479	.0124145	-.1512237	.0920987
R_BSESN	1,415	.0005519	.0111187	-.1410179	.085948
R_PSX100	1,415	.0002024	.0136201	-.0754066	.0750413
R_DSE	1,415	-1.02e-06	.0108496	-.144831	.1353414
R_ASPI	1,415	.000684	.0130644	-.1585994	.070842

Unit-Root (Stationarity) Testing

To ensure the sample mean and covariance matrix were stationary, which is a requirement for the Markowitz optimization, two tests were performed on each series at level and at first difference: the Augmented Dickey-Fuller (ADF) test and the Phillips-Perron (PP) test. As given in Tables 2 and 3, the null hypothesis of a unit root is rejected for nine of the ten series for level at the 1% significance level, indicating that the return series are stationary and $I(0)$, as expected for well-formed return data. The only exception is USDT; the null hypothesis of non-stationarity is not rejected by either test, at $p = .9939$ and $p = 1.0000$, respectively, and, surprisingly, in first differences as well ($p = .9947$ and $p = 1.0000$). The extreme outlier(s) from the rare de-pegging episodes of USDT have skewed the test statistics; therefore, USDT is retained in the descriptive and cointegration analyses with the caveat explicitly noted, and a sensitivity check excluding the de-pegging outlier is recommended as a robustness test.

Table 2. Augmented Dickey-Fuller (ADF) Unit Root Test Results

Variable	<i>t</i> -stat (Level)	Prob. (Level)	<i>t</i> -stat (1st Diff.)	Prob. (1st Diff.)
R_BTC	-16.739	0.0000	-21.860	0.0000
R_ETH	-17.486	0.0000	-21.036	0.0000
R_USDT	0.963	0.9939	1.045	0.9947
R_BNB	-16.881	0.0000	-21.102	0.0000
R_XRP	-22.753	0.0000	-22.167	0.0000
R_NSEI	-19.224	0.0000	-18.755	0.0000
R_BSES	-19.720	0.0000	-18.332	0.0000
R_PXS100	-16.679	0.0000	-15.563	0.0000
R_DSE	-19.138	0.0000	-23.396	0.0000
R_ASPI	-17.475	0.0000	-28.249	0.0000

Table 3. Phillips-Perron (PP) Unit Root Test Results

Variable	<i>t</i> -stat (Level)	Prob. (Level)	<i>t</i> -stat (1st Diff.)	Prob. (1st Diff.)
R_BTC	-32.270	0.0000	-47.465	0.0000
R_ETH	-33.987	0.0000	-49.375	0.0000
R_USDT	24.906	1.0000	7.638	1.0000
R_BNB	-33.064	0.0000	-50.650	0.0000
R_XRP	-32.267	0.0000	-43.645	0.0000
R_NSEI	-36.873	0.0000	-52.913	0.0000
R_BSES	-37.422	0.0000	-53.237	0.0000
R_PXS100	-27.937	0.0000	-41.159	0.0000
R_DSE	-38.960	0.0000	-48.793	0.0000
R_ASPI	-33.925	0.0000	-44.542	0.0000

Lag-Order Selection

The optimal number of lags of the VAR used for the cointegration test was evaluated by the standard information criteria (the sequential modified likelihood-ratio test (LR), the Final Prediction Error (FPE), the Akaike Information Criterion (AIC), the Hannan-Quinn Information Criterion (HQIC), and the Schwarz Bayesian Information Criterion (SBIC)) over lag orders 0 to 4. The two criteria, FPE and AIC, prefer a lag

length of 2. In comparison, the two criteria, HQIC and SBIC, which impose steeper penalties for adding more parameters and are generally preferred in large multivariate systems like the present ten-variable system, prefer a more parsimonious lag length, namely, 1. As is often done in an over-parameterized VAR system, this study uses the more parsimonious lag length suggested by the HQIC and SBIC criteria to prevent overfitting, while also reporting the

number of lags suggested by the FPE and AIC as a robustness alternative.

Table 4. VAR Lag Order Selection Criteria

<i>Lag</i>	<i>LogL</i>	<i>LR</i>	<i>FPE</i>	<i>AIC</i>	<i>HQIC</i>	<i>SBIC</i>
0	408.596	—	2.7e-14	−2.85732	−2.80511	−2.72717
1	6578.37	12340	3.5e-33	−46.3682	−45.7939*	−44.9366*
2	6749.66	342.58	2.1e-33*	−46.8793*	−45.7829	−44.1461
3	6836.16	173*	2.3e-33	−46.7825	−45.164	−42.7478
4	6889.92	107.52	3.2e-33	−46.451	−44.3104	−41.1148

*Indicates the lag order selected by the respective criterion.

Johansen Cointegration Test

To identify the number of long-run cointegrating relationships, the Johansen (1991) trace test was applied to the system of ten variables. The trace statistic is used to test the hypothesis of at most r cointegrating vectors, starting with $r = 0$, and the hypothesis of more than r cointegrating vectors. As shown in Table 5, the trace statistic exceeds its 5% critical value at $r = 0$ ($880.6873 > 233.13$) and at $r = 1$ ($205.7771 > 192.89$), leading to rejection of both null hypotheses, but falls below the critical value at $r = 2$ ($131.0058 < 156.00$), at which point the null hypothesis can no longer be rejected.

Thus, the test finds two cointegrating vectors ($r = 2$) from the ten cryptocurrencies and equity-index return series. The findings have direct implications for the theoretical framework: at least two long-run equilibrium relationships tie subsets of the series of cryptocurrencies and the equity series for the South Asian markets together, and therefore, the diversification benefit that can be derived from combining these assets may be more constrained in the long run than suggested by a short-run correlation analysis and warrants a vector error correction model rather than an unrestricted VAR.

Table 5. Johansen Cointegration Test Results (Trace Statistic)

<i>Max. Rank</i>	<i>Log-Likelihood</i>	<i>Eigenvalue</i>	<i>Trace Statistic</i>	<i>5% Critical Value</i>
0	75311.484	—	880.6873	233.13
1	75648.939	0.20637	205.7771	192.89
2	75686.325	0.02528	131.0058*	156.00
3	75703.147	0.01146	97.3613	124.24
4	75719.189	0.01093	65.2784	94.15
5	75730.448	0.00768	42.7586	68.52
6	75738.747	0.00567	26.1611	47.21
7	75743.449	0.00322	16.7578	29.68
8	75747.571	0.00204	8.5145	15.41
9	75751.077	0.00240	1.5012	3.76
10	75751.828	0.00051	—	—

*Indicates the cointegrating rank selected (first rank at which the trace statistic falls below its critical value).

Vector Error Correction Model (VECM)

Estimation

Since there are two vectors named “cointegrating,” a vector error correction model was estimated to model the short-run dynamics and long-run equilibrium adjustment behavior of the return

series. Fit statistics for each equation in the equation system are given in Table 6. The ten equations are, in all cases, highly statistically significant ($p = .0000$ for the equations based on the chi-square test for joint significance), suggesting that the short-run and error-correction

terms of the system, when considered together, explain a significant amount of variance in each of the equations. However, the magnitude of the explained variance (R^2) ranges widely from 27.0% in the case of $Dlnbtc$ to as little as 1.7% in the case of $Dlnnsei$, indicating that, in general, the short-run and error-correction dynamics of the cryptocurrency system explain, at best, a relatively small proportion of the variance in the returns of each of the equations, compared with the short-

run and error-correction dynamics of the equity-index equations. The VECM residuals and the coefficients of the confirmed cointegrating relationships are used in the optimization to verify that the sample covariance matrix is correct and to determine whether including the confirmed cointegrating relationships significantly alters the efficient frontier relative to that obtained from a naive, static covariance estimate

Table 6. Vector Error Correction Model – Equation Fit Statistics

Equation	RMSE	R^2	Chi ²	p-value
D_lnbtc	0.0114	0.2704	1076.84	0.0000
D_lneth	0.0319	0.0903	288.60	0.0000
D_lnusdt	0.2157	0.2028	739.36	0.0000
D_lnbnb	0.0334	0.0538	165.30	0.0000
D_lnxrp	0.0411	0.1460	496.62	0.0000
D_lnnsei	0.0098	0.0171	50.61	0.0000
$D_lnbsezn$	0.0087	0.0382	115.56	0.0000
$D_lnpsx100$	0.0110	0.0168	49.71	0.0000
D_lnlse	0.0090	0.0275	82.05	0.0000
D_lnaspi	0.0111	0.0251	74.96	0.0000

The Markowitz Mean-Variance Optimization, also known as the Portfolio Optimization Procedure, is one of the most popular optimization methods

After determining the time-series properties of the return series, the second methodological step is to use the Markowitz mean-variance optimization to construct the efficient frontier and select the optimal cryptocurrency-equity portfolio. The steps in the process are as follows:

1. **Expected return vector.** Each of the 10 assets has an annualized expected return calculated from the historical daily mean returns published in Table 1.
2. **Risk (covariance) matrix.** The variance-covariance matrix of the sample data of the 10 daily return series is estimated and annualized. It is a robustness check using information from the cointegration and VECM results. A second covariance estimate is calculated from the VECM residuals and compared to the unconditional sample covariance matrix, because two cointegration

vectors suggest that the unconditional covariance matrix may underestimate the long-run co-movement between some series.

3. **Optimization problem.** The classical Markowitz quadratic program is solved: minimize the variance of the portfolio $w^{\text{mar}} = w^T S w$, subject to a target expected return, $w^{\text{mer}} = R_{\text{tar}}^{\text{e}}$, followed by a long-only, no-short-selling constraint, $w \geq 0$, which reflects the practical limitation most retail and institutional investors in the South Asian region face when short-selling cryptocurrencies or the constituents of the equity indices.
4. **Efficient frontier construction.** This minimum-variance portfolio's efficient frontier is traced out by changing the target return parametrically.
5. **Portfolio of minimum variance (PV).** The portfolio is defined as the conservative benchmark allocation based on the lowest possible variance on the frontier, regardless of return.

6. **Optimal (tangency) risk-adjusted portfolio.** Using an appropriate risk-free rate proxy (short-term government treasury bills for the region) or an internationally accepted proxy (such as the US Treasury bill rate), the efficient frontier portfolio with the highest Sharpe ratio is determined as the optimal portfolio of cryptocurrencies and South Asian equity indices.
7. **Diversification and sensitivity analysis.** The possibility that the confirmed long-run cointegrating relationships lead to a considerable loss of the diversification benefits implied by the static model is explored by comparing the portfolio weights, expected return, standard deviation, and Sharpe ratio for the two optimizations.

Conclusion and Suggestions for Future Research

This study aimed to investigate whether diversification benefits, suggested by a short-run correlation structure, persist in a joint cryptocurrency (BTC, ETH, USDT, BNB, and XRP) and South Asian equity index (NSEI, BSESN, PSX100, DSE, and ASPI) portfolio when the time-series properties of the data are considered over the long period. The results provide a well-rounded response to the question of cross-asset-class diversification in this area and do not contradict the argument. The descriptive statistics also corroborate the risk asymmetry hypothesis that cryptocurrencies might offer higher average returns and volatility than South Asian equity indices, as they are a higher-risk, higher-reward asset class. However, USDT's behavior is quite different from the stablecoins' low-risk diversifier narrative, as its minimum return is extremely positive. It did not even achieve stationarity at the level or at first difference, which is due to rare de-pegging events, making its "stability" conditional rather than structural, and its inclusion in a mean-variance framework calls for caution and explicit robustness testing but not for assuming a constant, low risk like that of a fiat-pegged asset.

More important to the main research question of this study is the confirmation of two cointegration relationships apparent in the Johansen trace test

for the ten-return series. Theoretically, this is interesting because it shows at least two long-run equilibrium relationships tie subsets of cryptocurrencies and South Asian equities together, despite the short-run correlations (the quantity traditionally relied upon for diversification under static MPT) that might look low or unstable. That is, a short-run covariance-based portfolio may overestimate the diversification benefit that South Asian investors could realize, as the assets in the portfolio do not move independently but rather gravitate toward a long-run trend. This finding provides empirical evidence for the theoretical concern of diversification properties of cryptocurrencies with respect to traditional assets, as stated in Section 3, and reinforces the idea that not only the covariance matrix but also a vector error correction model is to be used in the optimization stage. The high significance of all the ten VECM equations, as well as the significantly higher explanatory power of the cryptocurrency equations compared to the equity-index equations, further supports the view that the short-run adjustment process of the cryptocurrency system is more closely linked to the system's overall movements than is the case with the South Asian equity-index system, with direct implications for the modeling of the contribution of the different cryptocurrencies to risk in a mean-variance optimizer.

Overall, these findings argue against using a naive, static Markowitz optimization to generate portfolio recommendations for South Asian investors seeking crypto exposure. Where the VECM-residual-based covariance matrix significantly differs from the unconditional sample covariance matrix, the efficient frontier, the global minimum variance portfolio, and the tangency portfolio derived from the two methods might suggest very different allocations—and the VECM-derived frontier, reflecting confirmed long-run equilibrium relationships, should be considered the more conservative and defensible allocation basis.

There are several constraints that lead to promising avenues for future studies. First, there is a need to conduct a separate sensitivity analysis

when USDT's de-pegging episodes are extreme outliers, to see whether the observed non-stationarity is a permanent characteristic of the series or simply a result of the individual crisis events, by winsorizing the series or removing the outlier periods. Third, this analysis should be expanded to explicitly estimate the pre-COVID, COVID, and post-COVID periods to test the robustness of the two cointegrating relationships found and to check whether the relationships themselves are time-varying. As shown in the literature reviewed in Section 2, the volatility and cointegration structure of cryptocurrencies are regime-dependent, especially during the COVID-19 pandemic. Third, the results of selecting the number of lags differed (FPE/AIC with 2 lags versus HQIC/SBIC with 1 lag); future investigations may include reporting and comparing portfolio results across both lag specifications as a further robustness check. Finally, incorporating other stablecoins, gold, or region-specific fixed-income products would further elucidate the nature of the asset-universe constraints discussed in the present paper and determine whether they are specific to the 5 selected cryptocurrencies or representative of a broader structural asset-universe constraint between digital assets and South Asian equity markets. Finally, the use of the mean-semi-variance approach with ANN as an illustrative example of machine-learning portfolio construction techniques that have been demonstrated to improve upon classical portfolio construction methods in South Asian markets, as suggested elsewhere in the literature, is a promising area for further development in order to outperform the classical Markowitz portfolio construction approach employed in this study. Overall, this study makes a significant contribution to the literature by using all of the rigorous stationarity, cointegration, and VECM diagnostic tests before optimizing a South Asian equity portfolio with cryptocurrencies. This methodology is missing in previous crypto-diversification studies based on MPT.

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