

COMPARATIVE INVESTIGATION OF VARIOUS POROUS FINS SUBJECT TO JOULE HEATING AND VARIABLE PROPERTIES: A CASE STUDY OF THERMAL EFFICIENCY

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Abstract

Heat exchangers are essential for removing heat from heated surfaces. In this regard rectangular fins with various shape being extensively used in engineering applications due to their effectiveness and straightforward design. By optimizing surface radiation and convection, these fins are designed to increase the efficiency of heat transfer from the primary heated surface. Therefore, this study examines the heat transfer and thermal efficiency through convective-radiative four different types of porous fin. Lorentz force is considered to assess four distinct longitudinal fins with thin-tip-exponential, trapezoidal, thick-tip-exponential and rectangular configurations. Variable surface emissivity is discussed in the presence of conduction, convection, and heat radiation transfer. Numerical solutions for modeled equations are often obtained using the ND-Solve technique after they have been transformed into dimensionless ODEs. The results show that convection and radiation have a major impact on the fins' thermal performance. Greater convective parameters lead to lower fin temperature whereas an increase in radiation parameters enhances the heat dissipation. Furthermore, the effect of magnetic fields on heat transfer efficiency shows that stronger magnetic field diminishes the temperature gradients. Overall analysis reflects that an exponential fin exhibits the best efficiency and temperature distribution compared with other ones.

1. Introduction

One of engineering's main goals has to improve heat transfer mechanisms, especially by using extended surfaces (fins) with various geometric patterns and cutting-edge cooling fluids like nano-fluids. A hot source can transmit more thermal energy to a cooler surrounding fluid than surfaces without fins. Fins significantly uplifted the surface area available for heat dissipation. Superheaters, automotive systems, electrical devices,

microprocessors, central processing units (CPUs), aircraft engines, gas turbines, refrigeration, automobile radiators, air conditioning, and other industrial and consumer technologies are just a few of the many applications where these improvements in heat transfer are essential. One of the most fascinating areas of thermal engineering is still the goal of increased heat transfer rates. The need to create heat management models that are not only more

effective but also lightweight, affordable, and dependable. Integrating fins with main surfaces is a very successful method of improving heat transfer because it greatly expands the surface area available for heat dissipation. By easily transferring thermal energy from the equipment to the outer fluid fins, which function as expanded surfaces, enable effective heat transfer. To maximize heat dissipation, fins are usually design of high-thermal-conductivity materials like copper and aluminum. They are widely used in equipment that needs effective convective heat transmission, including electrical transformers, internal combustion engines, microelectronics, spacecraft, and solar panels. The thermal performance of several fin geometries, such as rectangular, exponential, trapezoidal, and radial-shaped arrangements, has been extensively studied due to the widespread use of fins. Many technical and industrial sectors, such as air conditioning systems, microelectronics, the automotive industry, power plants, refrigeration units, consumer electronics, cryogenic systems, and aerospace engineering, rely heavily on these crucial components [1–3]. Akinshilo [4] conducted further research on heat transfer in straight fins, incorporating temperature-dependent thermal conductivity and internal heat generation, utilizing the Adomian decomposition method. The study emphasized that increasing thermos-geometric parameters and thermal conductivity at the fin base significantly enhances heat dissipation. Additionally, comparative evaluations of trapezoidal and exponential fin geometries revealed that exponential fins achieve superior thermal performance, exhibiting greater heat dissipation than trapezoidal fins, particularly in the presence of multiple nonlinear effects [5,6]. The thermal behavior of trapezoidal fins has been a focal point of several studies. Khani and Aziz [7]. Employed the homotopy analysis method to evaluate the exact thermal performance of trapezoidal fins. At the same time, Kang [8]. Investigated two-dimensional heat transfer through trapezoidal fins with varying base thicknesses. These studies found that increasing the fin-shape factor linearly reduces heat loss. Das [9]. Analyzed the temperature profile of

trapezoidal fins, emphasizing that fin size and thermal conductivity coefficients have a more significant impact on temperature transmission compared to other variables. Turkyilmazoglu [10]. Extended this research by examining thermal transmission through moving trapezoidal fins, and Onah et al [11]. Thermal efficiency of trapezoidal fins, concluding that variations in mass flow rate intensify thermal loss efficiency, is studied by Gireesha et al [12]. Further they explored the cooling capability of convective–radiative trapezoidal fins, noting that porosity and wetness significantly enhance their performance.

One important area of research is to maximize thermal transfer and efficiency in engineering systems by examining various fin profiles. For example, it has been demonstrated that ignoring thermal radiation in rectangular fins lowers their thermal performance, especially in materials like copper and aluminum. It has been shown that broad surfaces and fins with different shapes greatly improve thermal performance in cooling operations [13]. Comparative research has shown that, in comparison to annular fins, star-shaped fins have better heat transmission properties. Similarly, both experimental and analytical techniques have been employed to compare the performance of straight parallel plate fins to that of W-shaped fins [14]. Thermal performance of longitudinal fins with trapezoidal profiles and the heat loss via rectangular fins in the presence of forced convection and low Reynolds numbers have also been studied. Higher rates of thermal transmission on rectangular extended surfaces have been observed using complex methods such as electro hydrodynamics [15]. Additionally, the thermal aspects of straight fins that mix radiation and convection effects have been evaluated analytically using techniques like the variation of parameters methodology [16]. These evaluations highlighted the importance of fin design and material selection in maximizing heat transmission across a wide range of engineering applications [17]. It has been shown that the temperature dispersion across fin surfaces is greatly increased by increasing thermo-geometric parameters [18]. However, because of increased

heat transmission from the fin surfaces to the surrounding environment, the total temperature tends to drop as the radiation temperature rises. Akinshilo et al. [19] investigated heat transmission in conduction principles, especially through insulated brick walls in structures, using the Navier-Stokes equations and the principles of heat conduction. Their findings indicated that cooling systems enhance occupant comfort and improve energy efficiency, minimizing energy waste. Another study, too, used finite-volume numerical simulations to determine the energy costs related to heat transfer through the walls of gas-fired and water-tube boilers that were insulated with materials such as calcium silicate and fiberglass [20]. The results demonstrated that gas-fired boilers offer greater economic efficiency in operation compared to water-tube boilers. However, the use of low pour fuel oil in such systems leads to higher pollutant emissions and more significant environmental impact than diesel oil.

Radiative heat transfer, alongside convective heat dissipation, plays a crucial role in determining the overall thermal efficiency of fins, especially when radiative losses are comparable to natural convection. To achieve an accurate thermal performance assessment, it is essential to account for the combined influence of conduction, convection, and radiation. Particularly in systems where convective or radiative heat transfer coefficients are relatively low Experimental and numerical investigations conducted by Mueller and Abu Mulaweh [21] on fins subjected to both convection and radiation demonstrated that radiation contributes around 15%–20% of the total heat dissipation. Likewise, Chaddock [22] reported that polished aluminum fins experience radiative heat loss in the range of 10% to 20% of overall thermal dissipation. Furthermore, Edwards and Chaddock [23] observed that in cylindrical fins with a high surface emissivity (0.99), radiative heat loss can account for nearly one-third of the total heat transfer. Arslanturk [24] introduced a decomposition mechanism to evaluate the efficiency of radiative rectangular fins, while Torabi and Zhang [25] carried out an analytical study on the thermal analysis and effectiveness of

convective–radiative straight fins with different configurations, accounting for nonlinear effects. They further examined the thermal behavior of straight fins subjected to combined convection and radiation under varying profiles. Additionally, research has focused on optimizing T-shaped fins [26] to enhance thermal efficiency. The study also explored trapezoidal fins [27] under mutual irradiation, applying geometric optimization techniques to convective–radiative longitudinal fins, as demonstrated by Azarkish et al. [28]. Additionally, Hosseinzadeh et al. [29] examined the influence of hybrid Nano-fluids and radiative parameters on heat transfer through hollow fins, concluding that these factors have a crucial involvement in rising thermal features.

Magnetic fields can significantly influence heat transfer, especially in fluids that conduct electricity, such as liquid metals, ionized gases (plasmas), or nanofluids with magnetic or electrically conductive nanoparticles. This effect is typically studied in the field of magneto-hydrodynamics (MHD). The utilization of magnetic fields on porous fins has also been a subject of interest. Hoshyar et al. [30] employed the least squares method to study this effect, while subsequent work extended the analysis using the Adomian Decomposition Technique [31]. Oguntala et al. [32] further investigated the role of magnetic fields in enhancing the heat transfer efficiency of porous fins using the finite-volume approach. Their findings showed a significant improvement in the heat transport capacity of fins when the magnetic field strength was increased. Collectively, these studies emphasize the importance of considering radiative effects, geometric optimization, and external factors, such as magnetic fields, in designing and analyzing fins for enhanced thermal efficiency across various sectors [33–37]. The implementation of longitudinal fins has been considerably studied using iterative techniques, particularly when including internal heat generation and the effects of magnetic fields [38]. Sreedevi and Reddy [39] investigated the influence of Lorentz force on fins under different conditions, highlighting the interaction between magnetic fields and thermal

performance. Studies have frequently used the RK method to explore how magnetic fields and convection jointly affect radiating radial porous fins [40, 41]. Comparisons between fins with and without magnetic fields indicate that the presence of magnetic fields notably enhances heat transfer efficiency in porous fins. Furthermore, the thermal behavior of moving porous fins with various profiles—convex, parabolic, concave, and trapezoidal immersed in hybrid Nano-fluids has been analyzed [42]. The highest efficiency and heat transfer rates were observed in concave parabolic fins among the various types studied. Studies on moving fins also indicated that doubling the Peclet number led to a 7% rises in the thermal profile, thereby enhancing overall thermal performance [43].

1.1. Scope and novelty

The investigation of modern thermal management mechanisms is the emerging key element in various areas, like comparative studies between rectangular, exponential, and trapezoidal fins have also been conducted numerically to analyze the

thermal efficiency of both fin types, considering multiple nonlinearities and heat generation. Their findings indicated that trapezoidal fins and exponential fins generally outperform rectangular fins in terms of thermal efficiency. By introducing dimensionless parameters into the governing equations, the study derived numerical solutions using the shooting technique, comparing rectangular, exponential, and trapezoidal fins. Graphical representations of the thermal profiles and efficiency of both fin types provided insights into the effects of various parameters, aligning with physical interpretations. These findings underscore the importance of geometric design, material properties, and external factors like magnetic fields in optimizing fin performance for enhanced heat transfer applications.

2. Mathematical modeling

In this paper, here the longitudinal fin models with four shapes trapezoidal fin, exponential fin's ($v < 0, v = 0, v > 0$), and rectangular fin as presented in Fig.1(a-d).

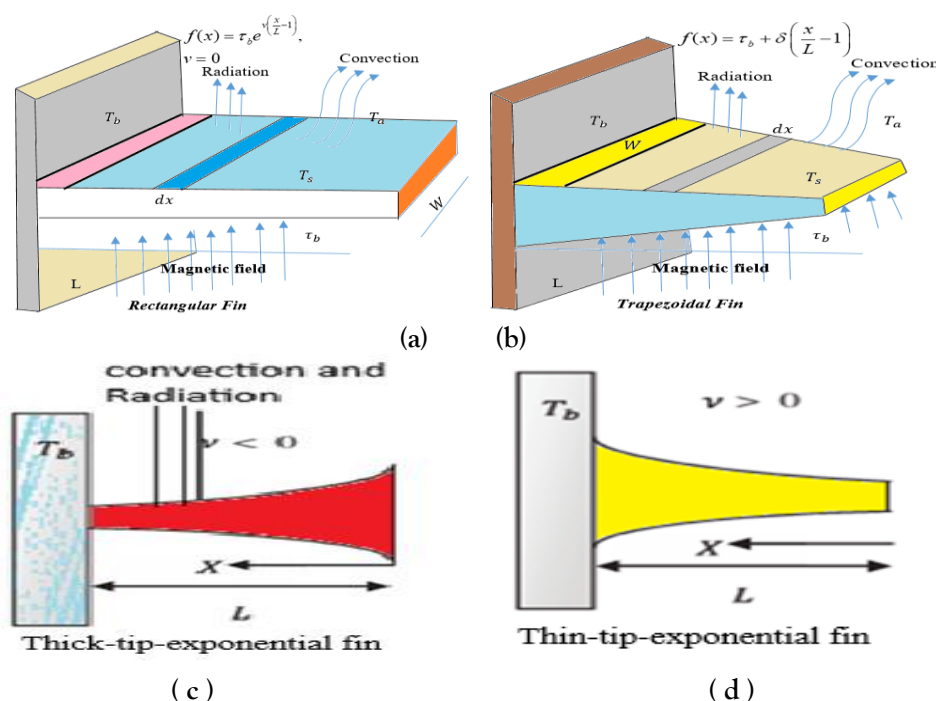


Fig.1 (a-d). Fin various geometrical configurations in the present analysis.

A mathematical model for this study is improved by using various fin forms.

Length (L) of a straight fin, T_b illustrates the heat extracted by the fin from the Base Surface, and T_a stand for heat transmission by Convection to the surroundings. T_s is the radiative heat transfer to a highly effective sink connected to the fin. The current conductivity (k) of the longitudinal fin. The convection heat transfer coefficient (h) and

emissivity (ϵ) of the fin's surface, respectively. Assuming that exterior emissivity (ϵ), transfer of heat coefficient (h), and thermal conductivity (k) vary with temperature, they can be expressed mathematically as given in equations (1), (2), and (3). These dependencies show a crucial role in accurately modeling the transfer of heat behavior in fins and other thermal systems.

$$h(T) = h_b \left[\frac{(T - T_a)^n}{(T_b - T_a)^n} \right], \quad (1)$$

$$k(T) = k_a [(1 + \phi)(T - T_a)], \quad (2)$$

$$\epsilon(T) = \epsilon_s [(1 + \phi)(T - T_s)]. \quad (3)$$

Where h_b represents the convection transfer of heat coefficient and k_a represents the transfer of heat through conduction at a given temperature T_a , $(T_b - T_a)$ is the difference in temperature ϵ_s is the emissivity at the fin's external with temperature T_s . The distinction in thermal

conductivity and emissivity on the external of the fin is scaled by dimensionless parameters that account for material properties and temperature-dependent effects ϕ and ϕ , respectively.

Under these assumptions, the energy equation that describes the fin's behavior is:

$$\left. \begin{aligned} q_x - (q_{x+dx}) + U \rho C_p f(x) \frac{dT}{dx} dx - \epsilon(T) \sigma P [T^4 - T_s^4] dx \\ - h(T) P [T - T_a] dx - \frac{j_c \times j_c}{\sigma} dx = 0, \end{aligned} \right\} \quad (4)$$

Consider a differential element of the fin with length dx shown.

Where q_x is heat in q_x at position x , q_{x+dx} is heat out q_{x+dx} at position $x + dx$ and heat loss by convection. $h(T)P(T - T_a)$

Where,

$$j_c = \sigma(E + V \times B),$$

(5)

Where, E stand for electric field, and $V \times B$ is the cross product of velocity and magnetic field

Taking $dx \rightarrow 0$ and simplifying Eq.(4) becomes for

$$\left. \begin{aligned} -\frac{dq}{dx} - h(T)P[T - T_a] + U \rho C_p f(x) \frac{dT}{dx} \\ - \epsilon(T) \sigma P [T^4 - T_s^4] - \frac{j_c \times j_c}{\sigma} = 0, \end{aligned} \right\} \quad (6)$$

According to Fourier's law transfer rate of heat through the fin is:

$$q = -f(x)Ak(T)\frac{dT}{dx}, \quad (7)$$

Putting equation (7) in equation (6) we obtain

$$\frac{d}{dx} \left[f(x)Ak(T)\frac{dT}{dx} \right] - h(T)P[T - T_a] + U\rho C_p f(x)\frac{dT}{dx} - \varepsilon(T)\sigma P[T^4 - T_s^4] - \frac{j_c \times j_c}{\sigma} = 0, \quad (8)$$

The longitudinal fin with a trapezoidal profile is characterized by its varying cross-sectional area along its length, designed to optimize heat dissipation efficiency.

$$f(x) = \tau_b + \delta \left(\frac{x}{L} - 1 \right), \quad (9)$$

Where τ_b indicates a base thickness that is semi-fin and δ shows fin taper.

Putting Eq. (1-3) and (9) in Eq. (8), one has.

$$\left\{ \begin{aligned} & k_a \frac{d}{dx} \left[(T - T_a)(1 + \phi) \left(\tau_b + \delta \left(\frac{x}{L} - 1 \right) \right) \frac{dT}{dx} \right] - h_a \left[\frac{T - T_a}{T_b - T_a} \right]^n [T - T_a] + \\ & U\rho C_p \left(\tau_b + \delta \left(\frac{x}{L} - 1 \right) \right) \frac{dT}{dx} - \left(\varepsilon_s [(T - T_s)(1 + \phi)] \right) \sigma [T^4 - T_s^4] - \frac{j_c \times j_c}{\sigma k_a A} = 0, \end{aligned} \right\} \quad (10)$$

The longitudinal fin with an exponential profile is characterized by a cross-sectional area that decreases exponentially along its length, optimizing heat dissipation by balancing conduction and convection effects efficiently.

$$f(x) = \tau_b e^{v \left(\frac{x}{L} - 1 \right)} \quad (11)$$

Now Eq. (8) is

$$\left\{ \begin{aligned} & k_a \frac{d}{dx} \left[(1 + \phi)(T - T_a) \left(\tau_b e^{v \left(\frac{x}{L} - 1 \right)} \right) \frac{dT}{dx} \right] - h_a \left[\frac{T - T_a}{T_b - T_a} \right]^n [T - T_a] + \\ & U\rho C_p \left(\tau_b e^{v \left(\frac{x}{L} - 1 \right)} \right) \frac{dT}{dx} - \left(\varepsilon_s [(1 + \phi)(T - T_s)] \right) \sigma [T^4 - T_s^4] - \frac{j_c \times j_c}{\sigma k_a A} = 0, \end{aligned} \right\} \quad (12)$$

Eq. (10) describes the local semi-fin thickness is a function of x ,

v is the shape factor and determines the exponential fin shape as

$v > 0$ resembles a thin-tip exponential fin

$v = 0$ resembles a rectangular fin

$$T(L) = T_b, \quad \left. \frac{dT}{dx} \right|_{x=0} = 0, \quad (13)$$

Also

$v < 0$ resembles a thick-tip exponential fin

Consider the boundary conditions where the fin tip is adiabatic, meaning no heat is lost at the tip, and the base temperature remains constant. Mathematically, these conditions can be expressed as:

$$\frac{j_c \times j_c}{\sigma} = \sigma_m B_o^2 u^2, \quad (14)$$

After using Eq. (14) into Eqs. (10) and (12) and considering the magnetic term as a linear function of temperature, we have.

$$\left. \begin{aligned} k_a \frac{d}{dx} \left[(1+\phi)(T-T_a) \left(\tau_b + \delta \left(\frac{x}{L} - 1 \right) \right) \frac{dT}{dx} \right] - h_a \left[\frac{T-T_a}{T_b-T_a} \right]^n [T-T_a] + \\ U \rho C_p \left(\tau_b + \delta \left(\frac{x}{L} - 1 \right) \right) \frac{dT}{dx} - (\varepsilon_s [(1+\phi)(T-T_s)]) \sigma [T^4 - T_s^4] - \frac{\sigma B_o^2 u^2}{k_a A} [T-T_a] = 0, \end{aligned} \right\} \quad (15)$$

and

$$\left. \begin{aligned} k_a \frac{d}{dx} \left[(1+\phi)(T-T_a) \left(\tau_b e^{\nu \left(\frac{x}{L} - 1 \right)} \right) \frac{dT}{dx} \right] - h_a \left[\frac{T-T_a}{T_b-T_a} \right]^n [T-T_a] + \\ U \rho C_p \left(\tau_b e^{\nu \left(\frac{x}{L} - 1 \right)} \right) \frac{dT}{dx} - (\varepsilon_s [(1+\phi)(T-T_s)]) \sigma [T^4 - T_s^4] - \frac{\sigma B_o^2 u^2}{k_a A} [T-T_a] = 0. \end{aligned} \right\} \quad (16)$$

Upon using Eqs. (15-16) in to Eq. (17) in view of following variable,

$$\left. \begin{aligned} X = \frac{x}{L}, \phi = \frac{T}{T_b}, \phi_a = \frac{T_a}{T_b}, \phi_s = \frac{T_s}{T_b}, \alpha = \phi T_b, \beta = \phi T_b, \gamma = \delta T_b, \\ N_c = \frac{h_a L^2}{k_a T_b}, N_r = \frac{\varepsilon_s \sigma L^2 T_b^3}{k_a T_b}, P_e = \frac{U \rho L}{\tau_b \lambda}, \Psi = \frac{T_b}{L}, H = \frac{\sigma_m B_o^2 u^2}{k_a A} \end{aligned} \right\} \quad (17)$$

Through the introduction of suitable dimensionless parameters and simplification, a generalized expression of the model heat equation can be derived for trapezoidal, exponential, and rectangular fins:

$$\left. \begin{aligned} \frac{d}{dX} \left[(1+\alpha)(1+\gamma)(\phi-\phi_a)(X-1) \frac{d\phi}{dX} \right] + p_e ((1+\gamma)(X-1)) \frac{d\phi}{dX} - \\ N_c (\phi-\phi_a)^n (\phi-\phi_a) - N_r [(1+\beta)(\phi-\phi_s)(\phi^4-\phi_s^4)] - H(\phi-\phi_a) = 0, \end{aligned} \right\} \quad (18)$$

$$\left. \begin{aligned} \frac{d}{dX} \left[(1+\alpha)(\phi-\phi_a) e^{\nu(X-1)} \frac{d\phi}{dX} \right] + p_e (e^{\nu(X-1)}) \frac{d\phi}{dX} - \\ N_c (\phi-\phi_a)^n (\phi-\phi_a) - N_r [(1+\beta)(\phi-\phi_s)(\phi^4-\phi_s^4)] - H(\phi-\phi_a) = 0, \end{aligned} \right\} \quad (19)$$

$$\left. \begin{aligned} \frac{d}{dX} \left[(1+\alpha)(\phi-\phi_a) \frac{d\phi}{dX} \right] + p_e \frac{d\phi}{dX} - N_c (\phi-\phi_a)^n (\phi-\phi_a) \\ - N_r [(1+\beta)(\phi-\phi_s)(\phi^4-\phi_s^4)] - H(\phi-\phi_a) = 0, \end{aligned} \right\} \quad (20)$$

with boundary condition is

$$\phi(1) = 1, \quad \phi'(0) = 0, \quad (21)$$

3. Efficiency

The concept indicates how well a fin transfers heat, expressed as the ratio of its actual heat transfer to the ideal scenario in which its entire area is at base temperature $Q_{actual} = -k_b(1 + \alpha)A_c \frac{(T_b - T_a)}{L} \left(\frac{d\phi}{dX} \right)_{X=1}$.

(22)

The ideal value of q can be computed once the entire fin has been brought to base temperature.

$$Q_{ideal} = \int_{A_s} \left[h(T_b)(T_b - T_a) + \sigma \epsilon (T_b^4 - T_s^4) + \frac{j_e \times j_e}{\sigma} \right] dA_s, \quad (23)$$

The ratio of actual to total heat transfer is the fin's efficiency. The fin efficiency can be expressed by the dimensionless quantities in the form.

$$\eta = \frac{Q_{actual}}{Q_{ideal}}, \quad (24)$$

$$\eta = \frac{-\left((1 + \alpha) \frac{d\phi}{dX} \right)_{X=1}}{\int_0^1 \left[N_c(1 - \phi_a) + N_r(1 - \phi_s^4) + H(1 - \phi_a) \right] dX}, \quad (25)$$

4. Results and discussions

Figure. 2 shows the temperature $\theta(x)$, across the fin for three different fin geometries i.e. rectangular fin (RF), trapezoidal fin (TF), and exponential fin (EF). The effects of two important parameters are examined in parts (a) and (b): (N_r) and (N_c), respectively. Figure. 2.(a). Plots temperature distribution at various values of the radiative parameter (N_r). The radiative parameter represents the ratio of the radiation heat transfer to conduction heat transfer. The higher the radiative parameter (N_r) is (blue to red to green), the lower is the temperature along the fin. This means that a larger radiative parameter will mean more heat will be transferred from the fin surface by the radiation. So, the temperature variation in fin is enhanced towards the surrounding and, consequently, the overall temperature profile is reduced. The temperature distribution in the exponential fin (EF) is always the highest whereas that of the rectangular fin (RF) is of next high and that of the trapezoidal fin (TF) is the lowest. This means that the geometry using the exponential fin is the most effective for retaining the heat, that is,

the trapezoidal fin is the most effective for the dissipating heat by combined conduction and radiation mechanism of the three fin geometries used. The points are likely experimental data or a set of data points, and the lines are presumably analytical or simulation results these are reasonably consistent with the experimental data. Figure. 2.(b). Plots the temperature distribution, depending on the Convective parameter (N_c). The convective parameter takes into account the amount of convective heat transfer between the fin and the Surrounding fluid. The Convective parameter (N_c) is increased from 0.4 (blue), to 0.6 (red) and to 0.8 (green) and the temperature profile along the fin decreases. This trend is expected because increasing the convective parameter will result in increased convection cooling of the surface of the fin towards the fluid. The fin will cool the air as the heat is extracted from it. As in Figure 2(a), the relative performance of the different fin geometries in terms of the temperature distribution is the same. The exponential fin (EF) seems to have the highest temperature, suggesting that it is not as good at

dissipating heat through convection as the other forms, or that it may be intended for higher heat retention. The rectangular fin (RF) is then followed by the trapezoidal fin (TF), which always has the lowest temperatures, indicating that the trapezoidal fin is the better fin with respect to convective heat transfer. The close association of the points with the continuous lines once again indicates the strength of the results presented here. In order to find out the temperature profile of the fins, the two types of heat transfer, namely, the radiative and convective heat transfer mechanisms assume great importance. As either the radiative or the convective parameter is raised, the fin temperature will decrease since the heat will be removed at a greater rate. The trapezoidal fin has the lowest temperatures for the 3 fin geometries: this indicates that the fin is more effective in transferring heat, while the exponential fin has the highest temperatures: this means the fin can be used in applications where it is required to retain heat, or where it is required to operate at high temperatures.

Exponential fin, rectangular fin, and trapezoidal fin are all considered and the $\theta(x)$, along the fin for each of these fin geometries is shown in Fig.3. The figure shows the effect of two different parameters; the surrounding temperature in part (a) and the surface temperature in part (b). The variation of dimensionless temperature of surroundings on the dimensionless temperature profile of the fin is shown in Fig.3(a). The convective heat transfer from the fin is greatly affected by the surrounding temperature. Generally, the temperature along the fin, $\theta(x)$, increases as the ϕ_a , increases from $\phi_a = 0.5$ (blue lines and points) to $\phi_a = 1$ (red lines and points) to $\phi_a = 1.5$ (green lines and points). This is a normal physical phenomenon: The higher the ambient temperature, the smaller the difference between the ambient fluid and fin, and the less the heat loss of the fin. As a result, the heat capacity of the fin is higher and its temperature is higher. The temperature distribution is always highest in the exponentially shaped fin (EF) and always lowest in the trapezoidal shaped fin (TF) with the rectangular shaped fin (RF) falling somewhere in

the middle. The exponential fin is the most effective fin in conserving the higher temperature while the trapezoidal fin is most effective in transferring heat to the surrounding under the influence of surroundings temperature, indicating less heat dissipation to the surroundings. The effect of the dimensionless surface temperature on the temperature distribution of the fin is shown in Fig. 3(b). Generally the surface temperature is the base temperature of the fin. The surface temperature ϕ_s , which is plotted along the lines and points of the red, green and blue bands, respectively, decreases as it is reduced from 1 to 0.8 to 0.5. Conversely, the higher the base temperature of the fin, the greater the temperature profile will be in the fin. This is a sensible thing to do since the temperature difference from base to tip of the fin gives a greater driving force for the heat to flow along the fin and then flow out to the surroundings, but it also increases the fin's temperature. As seen in Figure 3(a) as expected, the highest temperature distribution occurs for the exponential fin (EF), the lowest for the trapezoidal fin (TF) and intermediate for the rectangular fin (RF) for the same ϕ_s . This clearly indicates that the geometric arrangement plays a very important role on the heat transfer mechanism, with the trapezoidal fin configuration being the most efficient in terms of heat transfer and low fin temperature for the same boundary conditions. Temperature of surroundings and temperature of surface are important factors that can define the temperature profile of the fin. Generally, an increase in either of these temperatures results in a higher fin temperature. In both cases, the temperature distribution of the trapezoidal fin is the lowest and hence, has the best heat dissipation performance, while the temperature distribution of the exponential fin is the highest, and hence, the product can be manufactured with a higher operation temperature or lower heat loss. In all the plots, the continuous lines are well matched with the discrete points showing the results presented here to be more credible.

The $\theta(x)$, along the fin for three different fin geometries, namely trapezoidal fin (TF), rectangular fin (RF) and exponential fin (EF) is

shown by Fig. 4. In part (a) the effect of the Peclet number (Pe) is shown and the effect of the Hartmann number (Ha) is shown in part (b). Figure 4(a). Shows how the Temperature profile is altered when the Peclet number (Pe) changes. The Peclet number is a dimensionless number that is the ratio of heat transfer by convection to heat transfer by conduction. The temperature along the fin, $\theta(x)$, decreases as the Peclet number is increased from $Pe=0.2$ (blue lines and points) to $Pe=0.6$ (red lines and points) to $Pe=1$ (green lines and points). The trend is expected as convection is more dominant at higher Peclet number. The result is then that it is easier for the heat in the fin to be conveyed to the fluid surrounding the fin by convection, making the fin Temperature lower. The temperature distribution of the exponential fin (EF) is always the highest, followed by the rectangular fin (RF), and trapezoidal fin (TF) has the minimum $\theta(x)$, for any Peclet number. This implies that the exponential fin is less efficient in cooling through convection and the trapezoidal fin is more efficient in cooling by convection, thus lower Fin Temperatures. It is seen that the $\theta(x)$, of the fin changes with the Hartmann number (Ha) as shown in Figure 4(b). In magnetohydrodynamic (MHD) flows the Hartmann number is the ratio of body force of the electromagnetic field to the viscous body force. As the Hartmann number (Ha) is increased ($Ha=1$ (blue lines and points) to $Ha=2$ (red lines and points) to $Ha=3$ (green lines and points)), the temperature ($\theta(x)$) along the fin increases. The MHD heat transfer involves a stronger magnetic field, with increasing the Hartmann number. This increased magnetism produces a resistance that tries to put out any fluid movement, thus decreasing the fluid motion and the convection transfer and in turn the amount of heat transferred from the fin to the fluid. This means that the heat stays within the fin, giving a higher temperature gradient. The trapezoidal fin (TF) has the lowest temperature distribution while the highest temperature distribution is found for the exponential fin (EF) for any particular value of the Hartmann number (Ha), as can be seen in Figure 4(a), and the intermediate temperatures are always shown for the rectangular fin (RF). This

confirms the previous result that the trapezoidal fin has the highest heat dissipation rate amongst the other two examined, and the exponential fin has the lowest heat dissipation rate. The Peclet number and the Hartmann number play an important role in the temperature distribution of the fins. The higher the Peclet number the more convective heat transfer occurs and the lower the temperatures of the fins. On the other hand, stronger magnetic force corresponding to higher Hartmann number will suppress the convection and consequently higher fin temperature. In both cases, the $\theta(x)$, for the trapezoidal fin is the lowest, which means this fin has the best heat dissipation property, and the exponential fin has the higher $\theta(x)$, which means the fin has a better heat dissipation property under the condition of high working temperature and small heat dissipation. The results in all plots are well presented and in good agreement with both continuous lines and discrete points, thus increasing the reliability of the results.

Fig. 5 shows the fin efficiency, η , against radiation-convection number, Nr , for the three types of fins: exponential fin (EF), rectangular fin (RF), and trapezoidal fin (TF). The figure shows specific case studies of the influence of the (Ha) for two different values of n : $n=0$ in part (a), and $n=2$ in part (b). The fin efficiency versus the radiation-convection number (Nr) is plotted in Figure 5(a) for $n=0$. This value of n is probably associated with a specific case of heat transfer or fin geometry parameter such as simple power-law variation of fin thickness or a specific heat transfer correlation. It is observed that for all the fin types and Hartmann numbers, the efficiency of the fin (η) decreases as the radiation-convection number (Nr) increases. Usually the radiation-convection number is a composite of radiation and convection compared to conduction. An increase in Nr means that more heat will be lost from the surface of the fin; this is desirable if the heat is going to dissipate, but if the base heat into the fin is unchanged or if the efficiency of the fin in transferring the heat over its length decreases, the fin's performance will decrease. The greater the increase of the Hartmann number (Ha) for a

specific Nr , the higher the efficiency of the fin. This is unusual because one might think that the hindrance of free flow of the fluid would lead to decreased heat transfer. In other situations, however, an increase in Ha may result in a decrease in the convective losses at the surface of the fin by reducing the velocity of the fluid near the surface, or it may also be due to the ability of the fin to carry the magnetic field, and hence increase the overall efficiency. In this case the blue line corresponds to $Ha=1.05$; the red line to $Ha=1.15$ and the green line to $Ha=1.25$, for which the values of η are lowest, intermediate and highest respectively. The Efficiency of the (EF) is always the highest one followed by the (TF) and the (RF) with the lowest efficiency over the entire range of Nr and Ha . This indicates that for $n = 0$, the most efficient fin shape to transfer heat is the exponential fin and the least efficient fin shape to transfer heat is the rectangular fin. The fin efficiency variation with the radiation-convection number (Nr) for $n = 2$ is shown in Fig.5(b). This modification of the value of n is likely to be a different physical situation or fin shape that is different than part (a). As was true for part (a), the radiation-convection number (Nr) influences the efficiency of the fins (η) in a similar manner for all fin types and Hartmann numbers. As in part (a), changing the radiation-convection number (Nr) affects the efficiency of the fins (η) in a similar manner for all fin types and Hartmann numbers. The same principle is responsible for this, even better heat loss from fin surface. The fin efficiency increases with the increase of the Hartmann number Ha (Ha) as shown in (a). The lower the Ha (blue) the lower the η , the higher the Ha (red) the higher the η and the highest the Ha (green) the highest the η . Overall efficiency values and the size of this effect are typically greater than those in part (a) for similar conditions and a higher n value may result in greater efficiency for the fin under the same conditions. The relative efficiency of the fins is the same as in part (a): the EF is the most efficient, the TF is the next and the RF is the least efficient. The consistency means that these different geometries of fins have intrinsic properties of thermal performance for any value of

n or Ha . For both cases ($n = 0$ and $n = 2$) the conclusion was that the fin efficiency is always reduced for increasing Nr , i.e., for higher radiative and convective heat losses, the overall thermal performance is reduced as compared to the maximum of fin thermal performance, even though these heat losses are beneficial in cooling it. In contrast, the performance of the fins is always improved with the increase of the Hartmann number (Ha) representing that the magnetic field can be utilized to enhance the heat transfer from the fin, possibly by affecting the characteristics of the boundary layer or by reducing undesirable fluid flow. The efficiency of the exponential fin is higher than that of the trapezoidal fin and the rectangular fin, and is superior to the other two in heat transfer design parameter range studied. The trapezoidal fin is better than the rectangular fin. The graphical representation of the results is shown to be accurate and trustworthy as the continuous line matches the discrete points fairly well.

Figure 6 shows that the fin (η) declines as the Hartmann number (H) rises for all three types of fin (exponential fin (EF), trapezoidal fin (TF) and rectangular fin (RF)). The figure shows the effect of the (Nr) at $n=0$ in part (a) and $n=2$ in part (b). The figure 6(a) represents the efficiency of fins against the Hartmann number (Ha) for $n=0$. This value of n is probably a particular example of heat transfer or fin geometry parameter. The efficiency of the fins (η) decreases with increasing Hartmann number (Ha) for any fin type and radiation-conduction number. The Hartmann number is the ratio of electromagnetic body force to viscous force, and it's applicable to magnetohydrodynamic (MHD) flows. A higher H corresponds to a higher magnetic field which has the effect of dampening the fluid flow and reducing the convective heat transfer from the fin. This is because the reduced heat transfer from fin surface results in the decreased overall fin efficiency as a fin is unable to transfer heat to the surrounding fluid. Increasing the radiation-conduction number (Nr) for a given Ha results in a decrease in fin efficiency. This can be seen in the curves for efficiency of $Nr=0.91$ (blue), then for $Nr=0.90$ (red), and then for

$Nr=0.88$ (green) with the least efficiency. The radiation-conduction number represents the relative significance of the radiative heat transfer with respect to conduction heat transfer. Increase in Nr means greater radiative heat loss from the surface of the fin, thus greater total heat transfer from the fin, but it may decrease the overall efficiency of the fin as compared to that of the ideal fin or case where the fin is purely convective. The efficiency of the exponential fin (EF) is always the highest, the trapezoidal fin (TF) is the next and the rectangular fin (RF) is the lowest in all range of Ha and Nr values. This implies that the exponential fin design is the most efficient at transferring heat for $n=0$, and the least efficient is the rectangular fin design. Figure 6(b) shows the efficiency of fins as a function of the Hartmann number (Ha) for $n=2$. This change in " n " is probably a different physical condition or fin configuration than that of part (a). As in (a), fin efficiency (η) decreases with increasing Hartmann number (Ha) for all types of fins and radiation-conduction numbers. The basic reason is the same: the suppression of convective heat transfer is by the stronger magnetic field. As in part (a), an increase in radiation-conduction number (Nr) decreases the fin efficiency. The curves for $Nr=0.77$ (blue) have the highest efficiency, followed by $Nr=0.76$ (green) and the lowest efficiency is for $Nr=0.74$ (red). The relative efficiencies of the fins is the same as in part (a): The exponential fin (EF) is the most efficient, the trapezoidal fin (TF) is the second most efficient and the rectangular fin (RF) is the least efficient. Remarkably, the overall efficiency values in Figure 6(b) ($n=2$) are in general lower than in Figure 6(a) ($n=0$). This indicates that the thermal performance of the fin is greatly sensitive to the number of fins, n . In both cases ($n=0$ and $n=2$), the fin efficiency gets lower for higher Hartmann number (Ha). The primary one is the decrease in the convective heat transfer in the fin region due to the increase in the magnetic field. In the same way, higher radiation-conduction number (Nr) also leads to smaller fin efficiency, probably because of the increase of radiative heat loss which is added to the total heat

transfer, but that may not as effectively deplete the heat transferred as it would in ideal conditions. The exponential fin is well-designed for maximising heat transfer for the parameters under investigation, as seen by its higher efficiency compared to the trapezoidal and rectangular fins. Trapezoidal fin is better than rectangular fin because it has a larger area. The good agreement between the continuous lines and the discrete points is indicative of the validity and reliability of the results presented.

The fin efficiency (η) was plotted as a function of convection-conduction number (Nc) for three different fin geometries, namely, rectangular fin (RF), trapezoidal fin (TF) and exponential fin (EF) as shown in Figure 7. This figure considers the effect of the Thermal conductivity, α , at two values of n : (a) $n = 0$, and (b) $n = 2$. The results of the calculations for $n=0$ are plotted in figure 7(a) as the fin (η) against the (Nc). This value of n is probably related to a heat transfer case, or a fin geometry parameter. The fin (η) decreases with increasing (Nc) for all fin types and values of α . The convection-conduction number is the ratio of convective heat transfer to conductive heat transfer along the fin. An increase in Nc will increase the amount of heat lost from the surface of the fin by convection. This means that there's efficient cooling, but it also can result in a decrease in fin efficiency, since a higher percentage of the heat conducted into the fin base is lost rapidly near the base, minimizing the cooling effectiveness of the distal parts of the fin. For a specific number of fins Nc , increasing the thermal conductivity (α) will result in increasing the efficiency of a fin. This can be seen from the efficiency curve for $\alpha=0.92$ (red), $\alpha=0.91$ (blue) and $\alpha=0.90$ (green) where the red curve has the highest efficiency followed by the blue curve then the green curve with the lowest efficiency. The higher the thermal conductivity the better heat can be transferred from the base of the fin to its tip. As a result of this, all of the fin surface is utilized more effectively in dissipating heat and the overall fin efficiency is increased. Overall, the exponential fin (EF) is the most efficient followed by the trapezoidal fin (TF) and the rectangular fin (RF) is the least efficient for all the range of Nc and

α values considered. This implies that if $n = 0$, the exponential fin will be the most efficient in transferring heat and the rectangular fin will be the least efficient. The fin efficiency versus the convection-conduction number (N_c) for $n=2$ is shown in figure 7(b). The change of n is likely to be a different physical situation or a different fin configuration than part (a). As with part (a), an increase in the convection-conduction number, N_c , causes a decrease in fin efficiency, η , for all the fin types and values of α . The basic principle is the same: more convective heat transfer from the surface of the fin. As in part (a), the greater the thermal conductivity (α) of the fins, the more efficient the fins are. The highest efficiency corresponds to the curve of $\alpha=0.92$ (red) while $\alpha=0.89$ (green) has the lowest efficiency (blue). It appears that the curves have been moved around in the order for $\alpha=0.89$ and $\alpha=0.90$, compared to part (a) (green at the bottom and blue in the middle). This might indicate a slight change in the sensitivity to α for different values of n . The relative ranking of the efficiencies of the fins are the same as for part (a): the exponential fin (EF) is the most efficient, the trapezoidal fin (TF) is next, and the rectangular fin (RF) is the least efficient. The overall efficiency values in the above figure 7(b) ($n=2$) are generally lower than those in the above figure 7(a) ($n=0$) which is noteworthy. This shows that the parameter n has a significant impact on the thermal performance of the fin and that at $n=2$, the thermal performance of the fin was less than that of the fin at $n=0$ under similar circumstances. In either case ($n = 0$ or $n = 2$) the increasing (N_c) always results in a decline in fin efficiency, suggesting that higher convective heat losses will decrease the overall thermal performance. On the other hand, higher values of thermal conductivity (α) will always improve the efficiency of the fins since it will help to increase the heat transfer inside the fin. The (η) of the Trapezoidal and Rectangular fin is less than that of the Exponential fin which shows that the Exponential fin is the best fin design for the considered parameters that maximises heat transfer. The trapezoidal fin is better than the rectangular fin. The change in parameter n from 0

to 2 leads to a general decrease in fin efficiency, suggesting that the parameter n plays a crucial role in fin design and performance. The good fit of the continuous lines and the discrete points, reinforces the validity and reliability of the results presented.

Figure 8 illustrates the fin (η) as a function of the (ϕ_a) for three different fin geometries: (EF), (TF), and (RF). The figure examines the influence of the (ϕ_s) at two different values of n : $n = 0$ in part (a) and $n = 2$ in part (b). Figure 8(a) shows the fin efficiency against the (ϕ_a) for $n = 0$. This value of n likely corresponds to a specific case of heat transfer or a fin geometry parameter. For all fin types and values of ϕ_s , as the (ϕ_a) increases, the fin (η) declines. This is an expected outcome. A higher surrounding temperature reduces the temperature transformation between the fin and its environment. This smaller temperature difference diminishes the driving potential for heat transfer from the fin, thereby reducing its effectiveness and overall efficiency in dissipating heat. For a given ϕ_a , an increase in the (ϕ_s) leads to an increase in fin efficiency. This is evident as the curve for $\phi_s = 0.73$ (green) shows the highest efficiency, followed by $\phi_s = 0.71$ (blue), and then $\phi_s = 0.70$ (red) exhibiting the lowest efficiency. A higher surface (base) temperature typically means a larger temperature difference between the fin base and the ambient, which can enhance the overall heat transfer and thus potentially the fin efficiency, assuming other parameters remain constant. The (EF) consistently exhibits the highest efficiency, followed by the (TF), and then the (RF) showing the lowest efficiency across the range of ϕ_a and ϕ_s values. This suggests that for $n = 0$, the exponential fin design is the most effective in transferring heat, while the rectangular fin is the least efficient. Figure 8(b) presents the fin efficiency against the surrounding temperature (ϕ_a) for $n = 2$. This change in n likely represents a different physical scenario or fin configuration compared to part (a). Similar to part (a), an increase in the (ϕ_a) results in a decrease in fin (η) for all fin types and values of ϕ_s . The fundamental reason remains the same: a reduced temperature driving force for heat transfer. Consistent with

part (a), an increase in the (ϕ_s) leads to an increase in fin efficiency. The curve for $\phi_s = 0.269$ (green) shows the highest efficiency, followed by $\phi_s = 0.268$ (red and blue, indicating a very close or identical efficiency for these two values). This suggests that for $n = 2$, the sensitivity of efficiency to changes in ϕ_s might be less pronounced or different in magnitude compared to $n = 0$. The relative ranking of fin efficiencies remains consistent with part (a): the (EF) shows the highest efficiency, followed by the (TF), and the (RF) is the least efficient. It is noteworthy that the overall efficiency values in Figure 8(b) (for $n=2$) are significantly lower than those in Figure 8(a) (for $n = 0$). This indicates that the parameter n profoundly influences the fin's thermal performance, making the fins substantially less efficient at $n = 2$ compared to $n = 0$ under similar conditions. In both scenarios ($n = 0$ and $n = 2$), an increase in the (ϕ_a) consistently leads to a decrease in fin efficiency, as it reduces the temperature gradient driving heat transfer. Conversely, an increase in the surface (base) temperature (ϕ_a) generally enhances fin efficiency, as it provides a higher potential for heat dissipation. The exponential fin consistently

outperforms the trapezoidal and rectangular fins in terms of efficiency, indicating its superior design for maximizing heat transfer under the investigated conditions. The trapezoidal fin performs better than the rectangular fin. The significant decrease in overall efficiency when moving from $n = 0$ to $n = 2$ highlights the critical role of the parameter n in fin thermal design. The excellent alignment between the continuous lines and discrete points underscores the validity and reliability of the presented results. In conclusion, the choice of fin geometry profoundly impacts both the $\theta(x)$ and thermal efficiency. While the trapezoidal fin is effective for applications requiring lower fin temperatures, the exponential fin consistently proves to be the most efficient in transferring heat, making it a promising candidate for applications where maximizing heat dissipation per unit base heat input is critical, even if it operates at a slightly higher temperature. The influences of Peclet, Hartmann, radiative, convective, and temperature parameters are significant and must be carefully considered in fin design and optimization. Further investigation into the conflicting Hartmann number efficiency trends would be beneficial.

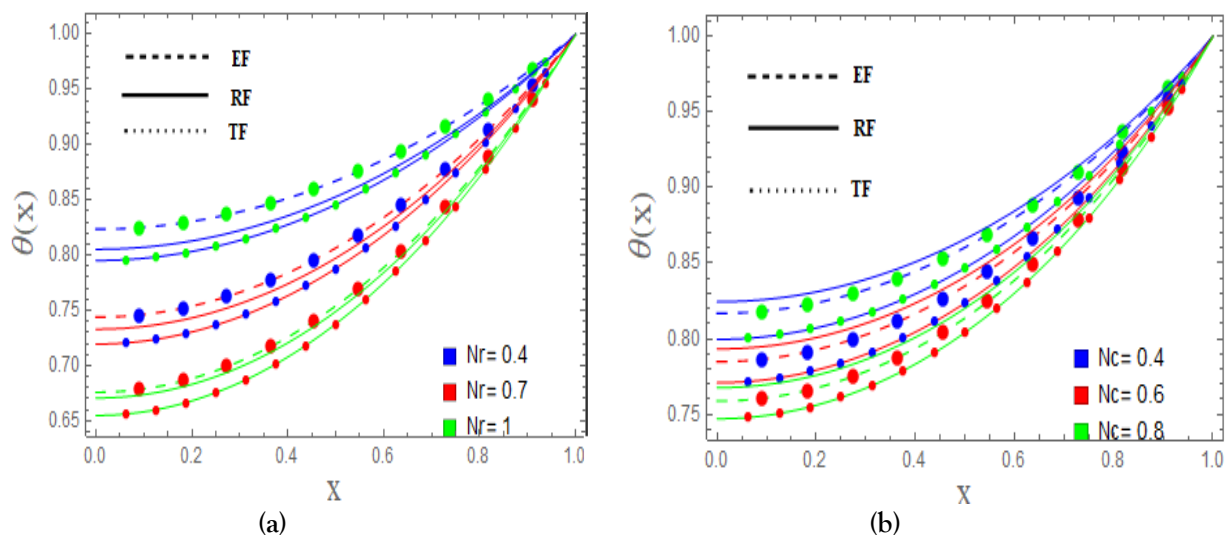


Fig.2. The $\theta(x)$ for Exponential, Trapezoidal, and Rectangular fins at the distinct values of (a).Nr. (b).Nc.

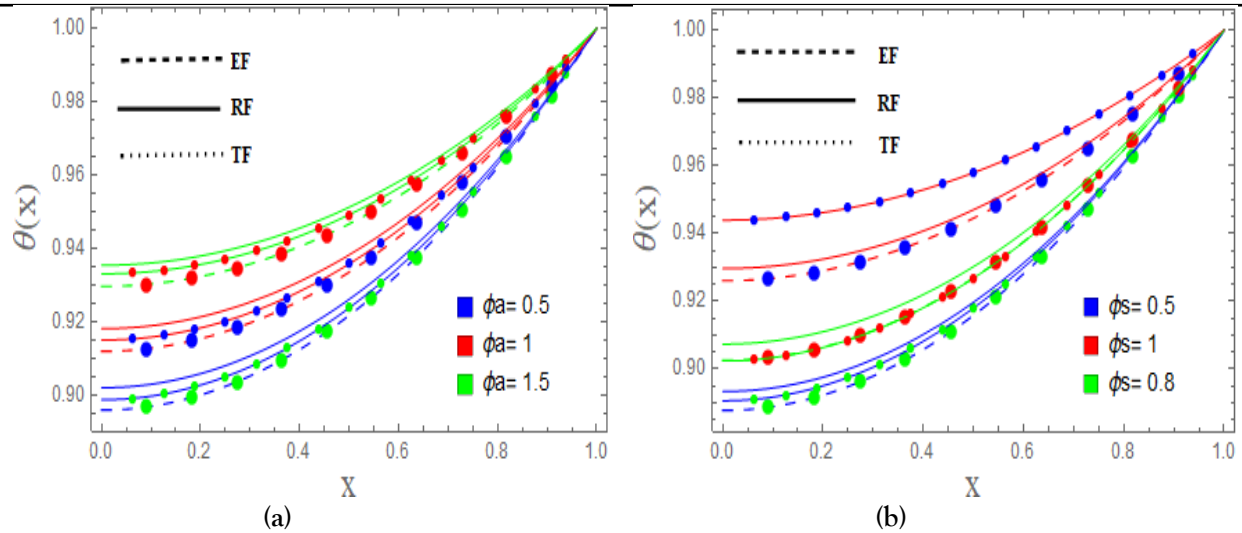


Fig.3. The $\theta(x)$ for Trapezoidal, Rectangular, Exponential and Trapezoidal fins at distinct values: (a) ϕa , (b) ϕs .

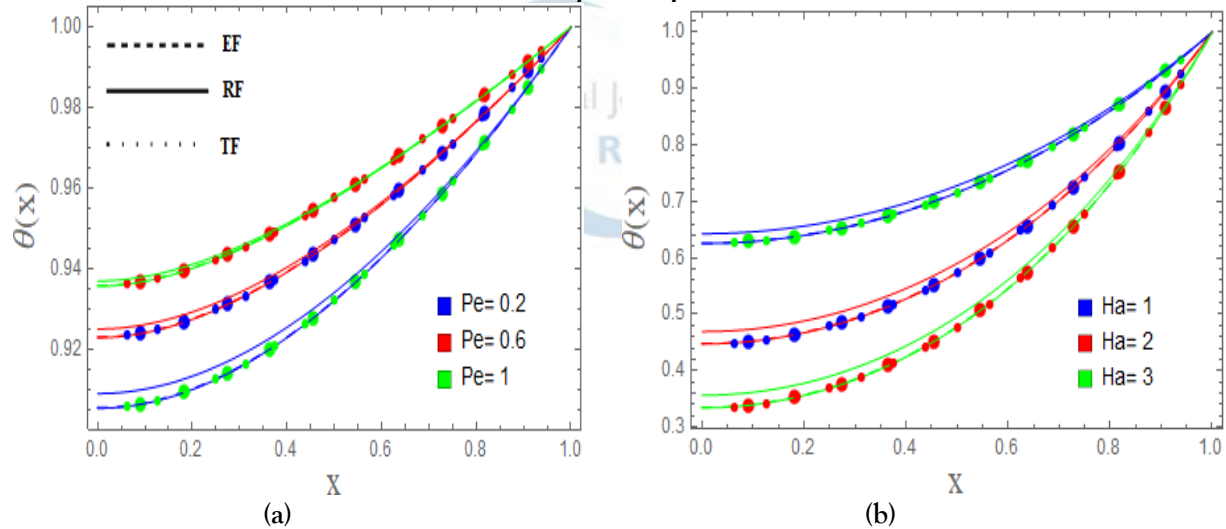


Fig.4. The figures illustrate the $\theta(x)$ for Trapezoidal, Rectangular, and Exponential fins: (a) (Pe), (b) (Ha).

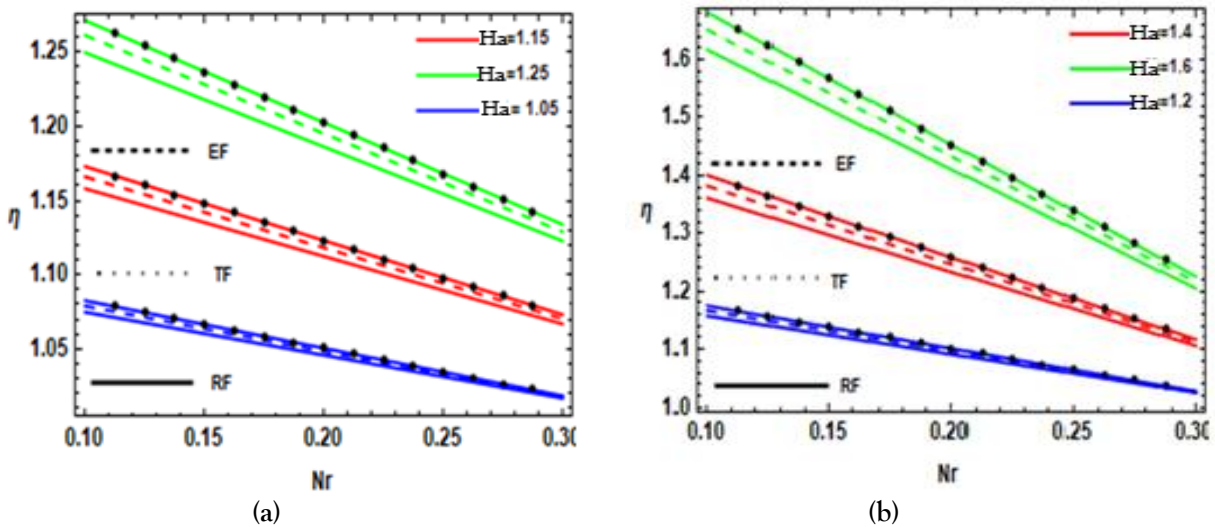


Fig. 5. Fin Efficiency vs. Radiation-Convection Number (Nr) under: (a) Hartmann number (Ha) influence, $n = 0$; (b) Hartmann number (Ha) influence, $n = 2$.

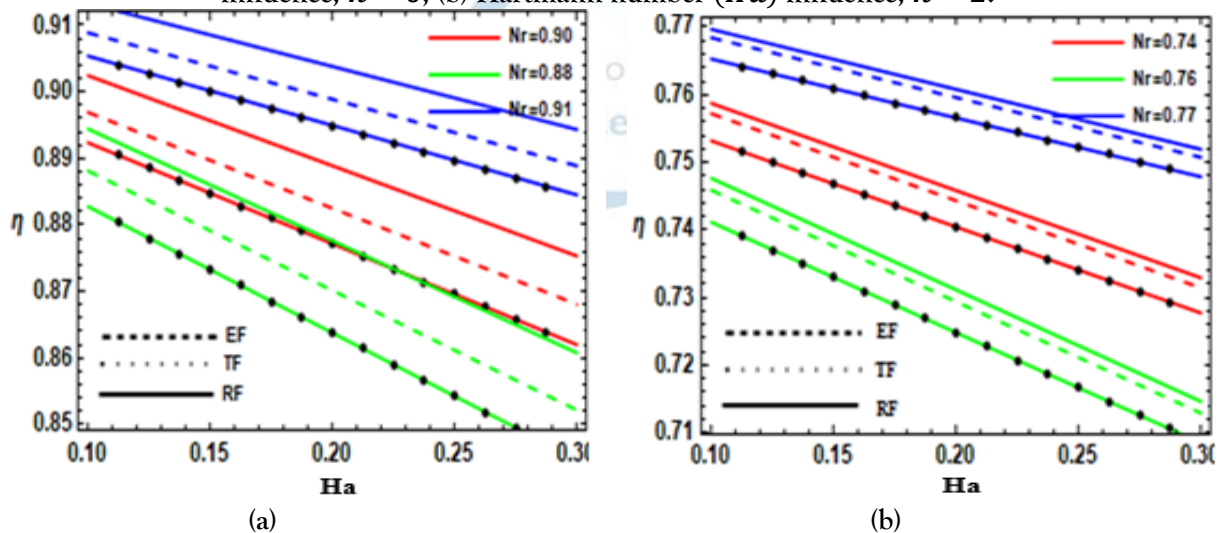


Fig.6. Fin's Efficiency against the Hartmann Number (Ha) under the Radiation-Conduction Number for (a) $n = 0$, (b) $n = 2$.

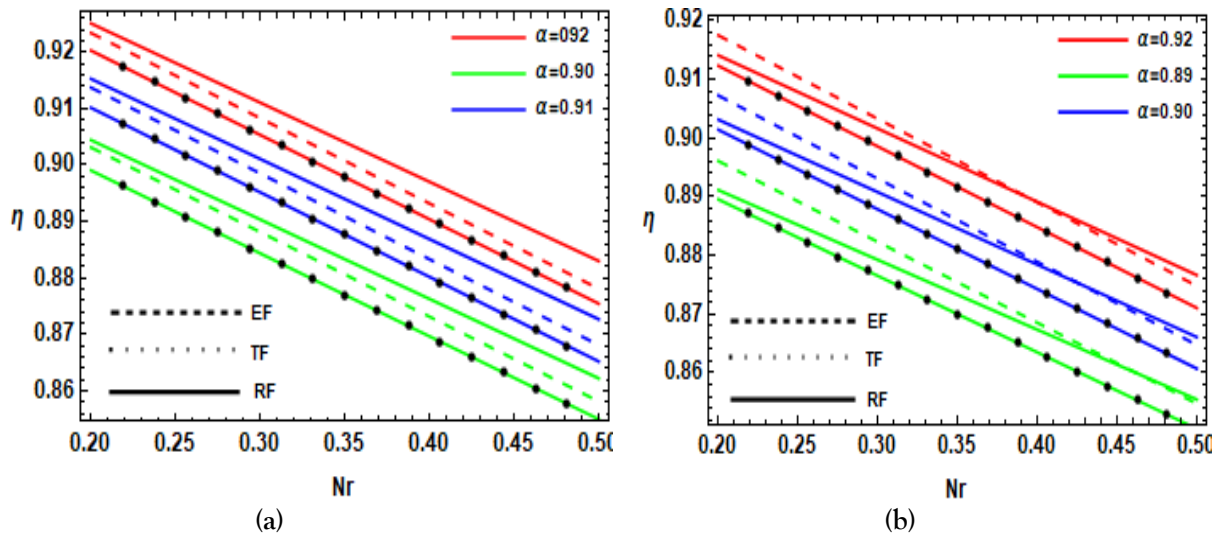


Fig. 7. Fin's Efficiency as a function of the Convection-Conduction Number (Nr) under the influence of Thermal conductivity, where (a) $n = 0$ (b) $n = 2$.

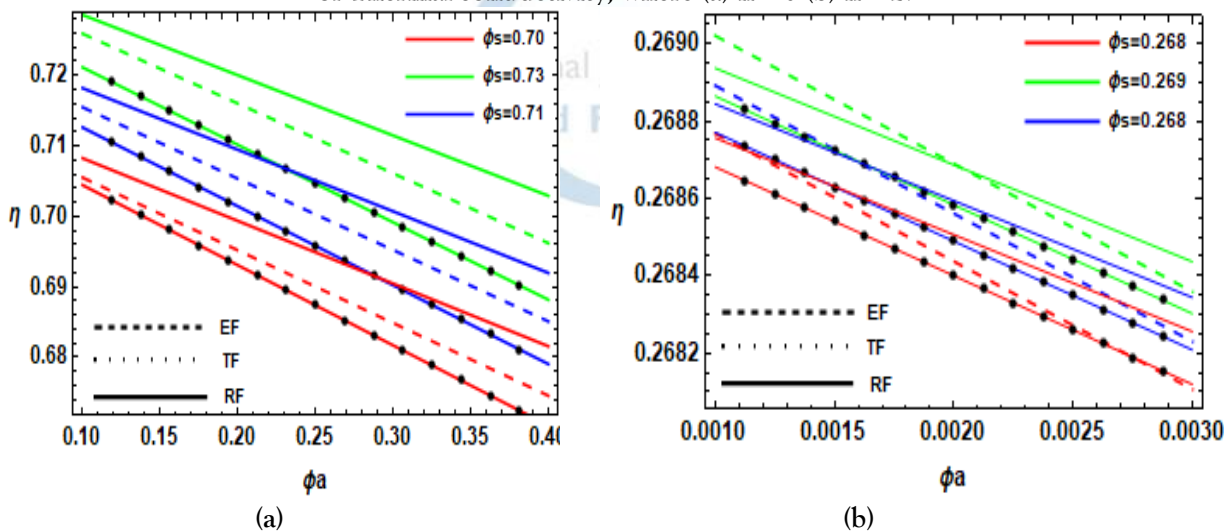


Fig. 8. Fin's efficiency as a function of surrounding temperature under the influence of surface temperature, where (a) $n = 0$, (b) $n = 2$.

5. Conclusion

Here, the Lorentz force and radiation features on heat transfer via four distinct shapes of porous fin with variable thermal conductivity are examined. Graphical representations are made using the computational technique to explore the thermal behavior of exponential, trapezoidal, and rectangular fins. The significant findings are listed as follows:

❖ Notably, exponential fins exhibit greater efficiency at lower radiative-conductive and

convective-conductive parameter values, which can be attributed to their variable cross-sectional area.

❖ By comparing different fin geometries, the study underscores the crucial role of shape in heat transfer efficiency.

5.1. Applications:

This analysis helps to mathematically optimize fin designs for better thermal performance in engineering applications. The gained knowledge

helps to improve passive cooling and to gain more knowledge about heat dissipation in different fin shapes. These insights offer practical suggestions for improving fin design in a variety of technical

applications, such as power electronics, cooling systems for automobiles, and thermal management in aircraft.

Nomenclature

A	Fin's surface area	α	Variable thermal conductivity parameter
h	Convection Heat Transfer Coefficient	β	Variable Surface Emissivity Parameter
k	Thermal Conductivity	γ	Fin Taper Ratio
L	Length (m) of Fin	∂	Base Thickness
n	Utilize transfer the heat coefficient power on the variable.	η	Fin's Efficiency
p	Perimeter (m) of Fin	λ	Thermal Diffusivity of Air
t	Fin's thickness m	φ	Thermal Conductivity Parameter
T	Dimensional Fin Temperature	θ	Dimensionless Local Temperature
U	Speed of moving Fin	θ_a	Ambient Temperature that is Dimensionless
W	Fin's width in (m)	θ_b	Dimensionless Base Temperature
x	x-axis direction in (m)	θ_b	Dimensionless Surface Temperature
X	Dimensionless Length	τ_b	Base Semi-Fin Thickness
E_f	Fin Effectiveness	ρ	Density of Material
C_p	Specific heat of the material	ϕ	Dimensionless Local Temperature
h_a	Fin's base Convection Heat Transfer coefficient	ϕ_a	Dimensionless Ambient Temperature
k_a	Thermal Conduction Convection Sink Temperature	Ψ	Aspect ratio
N_c	Convection -Conduction Number	ϕ_s	Surface Temperature that is Dimensionless
N_r	Radiation-Conduction Number	σ_m	Electric conductivity, [$\Omega^{-1} \cdot m^{-1} \cdot K^{-1}$]
P_e	Peclet Number	ε	Fin Emissivity
T_a	Ambient Temperature	ε_s	Temperature-dependent emissivity of the fin surface
T_b	Base Temperature	σ	Stefan-Boltzmann Constant
T_s	Surface temperature	u	Axial Velocity
B_0	Magnetic Field Intensity	j_c	Conduction Current Density
Ha	Hartmann Number	v	Fin Shape Parameter

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