

RECENT ADVANCES IN GRAPHENE PLASMONICS USING MAGNETIZED PLASMA WAVEGUIDES

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Abstract

Graphene is a promising material to develop small and reconfigurable photonic devices that is due to the excellent subwavelength electromagnetic confinement and electrical tunability. The external magnetic field results in the emergence of gyrotropic conductivity and the Hall response, and leads to more control over plasmon propagation and mode splitting, hybridization and nonreciprocal or chiral behavior. This review summarizes the basic properties of magnetoplasmons in graphene and their use in graphene-based waveguides, with a particular focus on the case of graphene circular geometry, which brings extra degrees of freedom in the modulation of all modes, due to curvature and angular momentum. The effects of magnetic field, chemical potential, waveguide radius, dielectric medium and material characteristics on mode confinement and propagation are taken into account. Various analytical and numerical techniques, such as electrostatic dispersion methods, plane-wave expansion, finite-element, transfer-matrix, boundary-element and time-domain methods, are reviewed to model these structures. Applications such as sensing, modulation, waveguiding, switching, resonators, and nonreciprocal photonic devices are suggested, and important challenges with regard to propagation loss, disorder, coupling, integration with magnetic fields, and accurate material modeling are pointed out.

1. INTRODUCTION

In Plasmonics, the interaction of electromagnetic radiation with the collective oscillations of free charge carriers at a material interface is taken into account. If these charge oscillations are coherently coupled to an electromagnetic field,

they are usually referred to as surface plasmon polaritons (SPPs). The electromagnetic energy of an SPP is confined near the interface, and consequently, the SPP wavelength is much smaller than the free space photon wavelength often. The plasmonics is thus a means to control

light at subwavelength scales, with potential applications in optical sensing, spectroscopy, communication, imaging and nanophotonic circuits [1]. For conventional plasmonic systems, noble metals like gold and silver have been extensively used because they have a negative real permittivity in the visible / near-infrared region which allows SPP modes to form. But, once the material and geometry is established, metallic plasmons are invariably accompanied by high ohmic losses, limited tunability, and relatively weak field control. Furthermore, although the frequencies of the optical and near-infrared (near-IR) are the most suitable for conventional metal plasmons, many emerging applications require dynamically tunable modes in the mid-infrared (mid-IR) and terahertz (THz) frequency ranges [2]. Graphene has been ushering a new platform to address these challenges. Graphene is a one-atom thick carbon crystal whose structure is a honeycomb lattice, and whose near-K and K' points of the Brillouin zone have linear relationship between energy and momentum. It conducts electricity like massless Dirac fermions with a Fermi velocity of $\sim 10^6 \text{ m s}^{-1}$ with the Fermi level being shifted by electrostatic gating, chemical doping, or electrical biasing [3]. Graphene is an appealing material for plasmonics to make a combination of its 2D nature, high carrier mobilities, broadband optical response, and powerful electrostatic tunability [4]. In particular, graphene can support transverse-magnetic plasmon polaritons in the THz and mid-IR frequencies, where the conventional noble-metal plasmons are less efficient. In 2012, near-field nano-imaging of infrared light was used to show for the first time that propagating graphene plasmons could have a controllable wavelength and amplitude, by tuning the gate voltage [5]. The intraband optical conductivity of the doped graphene can explain the physical origin of graphene plasmons. If the interband transitions are Pauli blocked, the conductivity is close to being Drude-like and is very sensitive to the chemical potential, the rate of scattering of the carriers, the temperature, and the frequency [5]. The plasmon dispersion in the electrostatic or

deeply sub-wavelength limit can be written as an approximate:

$$q_p \approx \frac{2\pi\epsilon_0\bar{\epsilon}\hbar^2\omega^2}{e^2E_F} \quad (1)$$

Where q_p is plasmon wave vector, ω is the angular frequency, e is the elementary charge, E_F is the Fermi energy, and $\bar{\epsilon}$ is an effective dielectric environment of graphene. The plasmon frequency is proportional to roughly, $\omega_p \propto \sqrt{E_F q_p}$. The dispersion is wave-vector dependent, which is quite different from the three-dimensional metals' bulk plasma response. It also helps to understand why graphene plasmons can be very strongly confined in space: at a given frequency, the wavelength of the plasmon in the graphene can be tens or even hundreds of times smaller than the free space wavelength of the same frequency. In such confinement, the density of optical states is increased and the light-matter interaction increases, which makes graphene very interesting for implementing compact waveguides, modulators, detectors, absorbers, sensors and nonlinear optical devices [6].

The tunability of graphene is especially important for the design of waveguides. The plasmon wave vector is dependent on E_F , so that the effective modal index, propagation constant, confinement and impedance of a graphene-based waveguide can change when the gate voltage is varied [1]. This structure can also be reconfigured after fabrication, as opposed to passive metallic waveguides, where the mode characteristics are mostly governed by the geometry and material composition. Graphene's atomically thin structure also opens the door to integration with other two-dimensional structures such as dielectric waveguides, microstructured substrates, metasurfaces, polar crystals, and other two-dimensional materials, without significantly increasing thickness. But this benefit comes with a set of problems such as imperfect coupling from free space, carrier scattering, disorder caused by the substrate, propagation length, fabrication nonuniformity, and the sensitivity of the plasmon response to the surrounding dielectric environment [7].

In the presence of a static magnetic field, an extra degree of freedom is introduced. If the magnetic bias is absent, an isotropic graphene sheet has an in-plane conductivity which is given as a scalar, $\sigma(\omega)$. If a magnetic field is applied perpendicular to the graphene plane, the Lorentz force will deflect the motion of charge carriers, which will break the time-reversal symmetry. The conductivity is then gyrotropic, and is expressed by a tensor with the form:

$$\sigma(\omega, B) = \begin{pmatrix} \sigma_d & \sigma_H \\ -\sigma_H & \sigma_d \end{pmatrix} \quad (2)$$

Where, σ_d is the diagonal or longitudinal conductivity and σ_H is the Hall conductivity. The off-diagonal Hall component is responsible for magneto-optical properties, including circular dichroism, Faraday rotation, cyclotron resonance and nonreciprocal wave propagation [8, 9] and couples the orthogonal components of the electric field. In the semiclassical regime, the cyclotron frequency of graphene not only depends on the magnetic field B but also on the carrier density or Fermi energy. The cyclotron frequency for a Dirac carrier is about:

$$\omega_c \approx \frac{eBv_F^2}{E_F} \quad (3)$$

Here B is the perpendicular magnetic flux density, This means that graphene can be simultaneously electrically and magnetically controlled on their magneto-optical properties [10].

When graphene plasmons interact with cyclotron motion, the collective charge oscillations appear and are called "magnetoplasmons". The dispersion is controlled by a combination of Coulomb restoring forces, carrier inertia, magnetic deflection, dielectric screening, and dissipation [11]. The hybridization of the ordinary plasmon frequency and the cyclotron frequency is observed in the magnetic field and described in a simplified way by the presence of mode shifts and polarization splitting of the modes in the magnetic field. More stringent treatments involve the full Kubo conductivity tensor, which is needed when there are interband transitions, Landau quantization, finite temperature, disorder, and quantum Hall effects. In some cases, the frequency, field profile,

polarization, group velocity and propagation direction of the resulting modes can deviate significantly from the zero-field graphene plasmons [12].

A major effect of magnetic bias is "nonreciprocity". The dispersion relation of a reciprocal waveguide is symmetric with respect to the change of the propagation wave vector direction, that is, $\omega(q) = \omega(-q)$. A perpendicular magnetic field is the key to break this symmetry by means of the Hall conductivity, so that the forward and backward propagating modes have different propagation constants, confinement factors, losses or even field distributions. The magnetic field can lead to one-way or highly-unbalanced propagation of plasmons in appropriate geometries. This behavior is especially relevant for integrated photonic systems, as it can lead to compact isolators, circulators, gyrators, directional couplers, reconfigurable mode switches and topological waveguides, rather than solely based on bulky magneto-optical materials.

Continuous and patterned graphene are both reported and studied to exhibit magnetoplasmonic properties. Experiments using electrical gating of graphene in the presence of a magnetic bias have shown that the frequency and strength of the magneto-optical resonances in the terahertz regime can be tuned by the carrier density [10]. Importantly, the response can be tuned by switching the polarity of the doping (electron or hole) at a fixed magnetic field, denoting the ambipolar nature of graphene and the change of the cyclotron response between electrons and holes [13]. By patterning graphene into antidot arrays, nanoribbons, disks or periodic plasmonic crystals, new resonances are added that are related with geometric confinement, Bragg scattering and edge magnetoplasmons. These can be used to achieve magneto-optical operation at higher frequencies and to offer yet another way to control the modal spectrum by varying the lattice period, feature size, filling factor and edge morphology [14].

The graphene waveguides are particularly promising for investigations of magnetoplasmons

due to the fact that they possess a strong two-dimensional confinement in combination with externally controllable gyrotropy. A waveguide can be created using a single sheet of graphene sandwiched between two dielectric materials, a graphene ribbon, two closely spaced graphene layers, a graphene-filled dielectric channel, or a patterned graphene metasurface. The guided modes in each case depend on the electromagnetic domain conditions and the conductivity tensor of the graphene [15]. The number, cutoff conditions, modal confinement, attenuation and coupling of modes may be changed by the magnetic field. In both ribbon and edge geometry, the Hall response can also concentrate modes around the physical edge, generating chiral edge magnetoplasmons with the direction of which depends on the magnetic field and carrier type. The modes of interest are these because they could localize the radiation along the edge of a graphene structure, thus minimizing radiation leakage and offering direct radiation transport along the perimeter of the structure [16].

There are many challenges to the theory and experiment of magnetized graphene waveguides. First, they are very sensitive to the conductivity model. A local Drude approximation can be used correctly in long wave length and highly doped graphene; however, it may be incorrect at high magnetic fields, near the interband thresholds and near the charge neutrality point and for plasmon wave vector which is large enough in which nonlocal effects become relevant. Second, in many cases the range of the magnetic field necessary to obtain an effective Hall response is rather large, particularly for medium mobility or high disorder samples. Thirdly, propagation of plasmons is restricted by scattering due to impurities, phonons, defects, edge roughness, interaction with the substrate, and spatial variations of the chemical potential. These losses can lead to resonances broadening and propagation length being limited even for high theoretical confinement. Fourth, the integration with the conventional silicon photonics may be difficult if external magnets or electromagnets are

used, and may lead to an enlarged device, higher power consumption, and higher cost of the system [17, 18]. Hence, the type of substrate and the encapsulation is the key to the performance of magnetized graphene waveguides. Charged impurity scattering, surface roughness, and spatial charge inhomogeneity may occur in graphene on conventional oxide substrates. Hexagonal boron nitride can be used for encapsulation which can significantly enhance mobility and decrease disorder due to its atomically flat surface and favorable dielectric environment [7]. However, hexagonal boron nitride (hBN) can give rise to its own resonances of phonon-polaritons, which can result in hybrid plasmon-phonon modes and other dispersion features. When designing realistic devices, the coupling between graphene plasmons and magnetic-field-induced cyclotron motion, between substrate phonons and plasmons, and between the geometry of the waveguides should be taken into account [14]. This review aims at discussing the basic concepts, difficulties and perspectives of plasmon modes in magnetized graphene waveguides. Plasmonic modes in magnetized graphene waveguides, from fundamental physics to applications and future prospectus: a schematic conceptual framework and research pathway is illustrated in **Figure 1**.

2. PLASMON MODES IN MAGNETIZED PLASMA

2.1 Characteristics of Magnetized Plasma Modes

A magnetized plasma is a medium of charged particles in which the magnetic field is a static external magnetic field, $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$. When there is no magnetic bias, the collective motion of the charge carriers is dominated primarily by the electric restoring force due to charge-density fluctuations. These oscillations lead to the generation of plasmons with dispersion that depends on the carrier density, the effective mass, and the wave-vector dependence of the Coulomb interaction, and the dielectric environment. In the presence of a magnetic field, the Lorentz force on the carriers ($\mathbf{F}_L = -e \mathbf{v} \times \mathbf{B}_0$) is added to the transverse force which causes motion of the carriers, and couples the longitudinal and

transverse components of current. Thus the optical response is now anisotropic, and can no longer be modeled with a scalar conductivity or

permittivity. Instead, the electromagnetic response is described by a tensor with longitudinal and Hall terms [19, 20].

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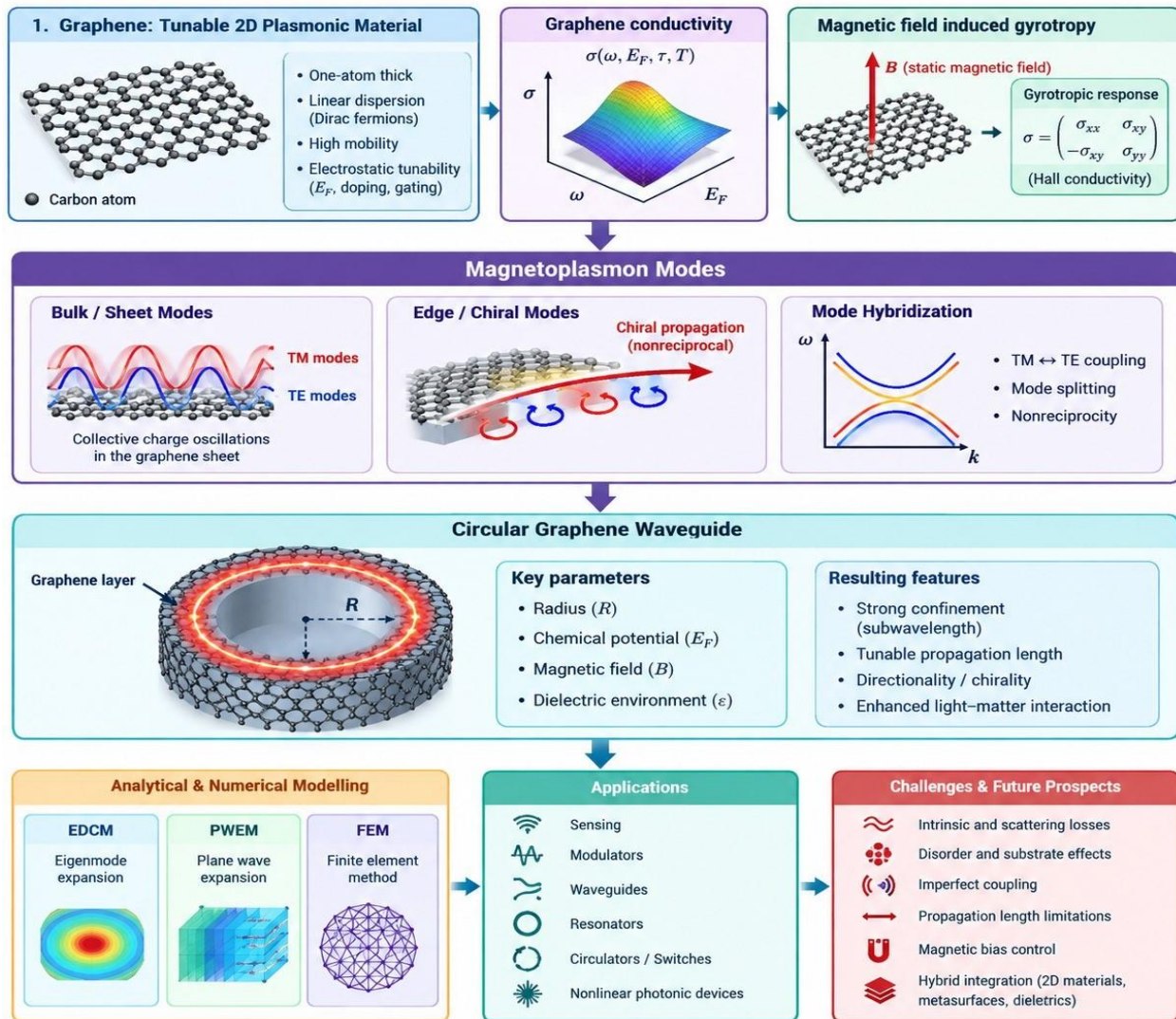


Figure 1: Conceptual framework of plasmonic mode in magnetized graphene waveguide.

For a two-dimensional electron system in the xy plane, the conductivity tensor may be written as

$$\sigma(\omega, \mathbf{B}_0) = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix} \quad (4)$$

Where, σ_{xx} is the parallel response to the applied electric field, and σ_{xy} is the Hall response. The off-diagonal element is odd with respect to magnetic-field reversal,

$$\sigma_{xy}(\omega, \mathbf{B}_0) = -\sigma_{xy}(\omega, -\mathbf{B}_0) \quad (5)$$

Where, the longitudinal component is generally even in the magnetic field. This antisymmetric Hall term is the basis of gyrotropy, polarization conversion, Faraday rotation, circular dichroism, and nonreciprocal propagation of plasmons in magnetized two-dimensional materials [10, 21]. For the conventional classical plasma, the characteristic frequencies are the plasma frequency and the cyclotron frequency. The

plasma frequency is the natural frequency of collective charge oscillations, and the cyclotron frequency is the frequency of the circular motion due to the magnetic field. The cyclotron frequency of a carrier with charge $-e$, effective mass m^* and magnetic field B_0 is: $\omega_c = \frac{e B_0}{m^*}$

Hybrid magnetoplasmon modes are the result of the interaction between plasma oscillations and cyclotron motion. The simplest long-wavelength model has an approximate bulk magnetoplasmon frequency:

$$\omega_{mp} \approx \sqrt{\omega_p^2 + \omega_c^2} \quad (6)$$

Herein, ω_p is the zero field plasma frequency. This expression is important because it reveals the general behavior of a bulk Plasmon's resonance frequency in an applied magnetic field, namely an increase in resonance frequency from the normal value, due to the cyclotron restoring contribution. In two-dimensional systems, however, the exact spectrum is more complex because ω_p is wave-vector dependent and because boundary conditions, dielectric screening, retardation, and Hall coupling strongly influence the mode structure [22].

In magnetized plasma, the waves that are propagated perpendicular to the magnetic field are usually referred to as ordinary modes and extraordinary modes. In the geometry considered the electric field component of the ordinary mode is not affected by the Lorentz force and is thus similar to the electric field component of unmagnetized plasma. In the extraordinary mode, the electric field is perpendicular to the magnetic field and the carrier gyromotion has large influence. It has magnetic field-dependent resonances and cutoffs such as upper-hybrid like behavior. While it has its roots in bulk plasma physics, the same physical separation is applicable to understanding magnetoplasmonic excitations in two dimensional materials. For graphene, however, the response is limited to an atomically thin layer, and the modes are more properly thought of as hybrid electromagnetic excitations characterized by a surface conductivity tensor,

instead of a three dimensional dielectric tensor [3].

2.2 Conductivity Tensor and Magnetoplasmon Formation in Graphene

Graphene provides an especially rich magnetoplasmon platform because its carriers possess a linear Dirac dispersion. The magnetic response is therefore not determined by a fixed effective mass alone. In the semiclassical regime, the cyclotron mass is approximately $m_c = E_F/v_F^2$. The graphene cyclotron frequency can be determined by equation (3) which shows that the magnetic response can be tuned electrically through the chemical potential. At lower carrier densities, the cyclotron frequency becomes larger for a fixed magnetic field, while electrostatic gating modifies both the zero-field plasmon frequency and the cyclotron response [23]. Graphene therefore offers two independent control parameters: the carrier density controls the longitudinal Drude response, and the magnetic field controls the Hall response and time-reversal-symmetry breaking [24].

In the semiclassical Drude approximation, the magneto-optical conductivity of doped graphene may be represented as:

$$\sigma_{xx}(\omega) = \frac{\sigma_0(1-i\omega\tau)}{(1-i\omega\tau)^2 + \omega_c^2\tau^2} \quad (7)$$

and

$$\sigma_{xy}(\omega) = -\frac{\sigma_0\omega_c\tau}{(1-i\omega\tau)^2 + \omega_c^2\tau^2} \quad (8)$$

Here, σ_0 is the zero field Drude conductivity and τ is the momentum-relaxation time. In the Kubo formalism, equivalent expressions can be obtained when the interband transitions, Landau level quantization, temperature, disorder and quantum Hall effects become relevant. In the basis of circular polarization the tensor is diagonalized as follows [1, 5, 25]:

$$\sigma_{\pm} = \sigma_{xx} \pm i\sigma_{xy} \quad (9)$$

The two circularly polarized conductivities are for the motion of the carriers which is clockwise and counter clock-wise with respect to the magnetic field. These two rotation senses are quite different in the magnetic field, so the two circular polarizations are no longer degenerate. This produces mode splitting, polarization-selective

absorption, and different propagation constants for opposite circular polarizations.

For a graphene sheet located between two dielectric media, the plasmon modes are obtained by applying Maxwell's boundary conditions together with the surface-current relation:

$$\mathbf{J}_s = \boldsymbol{\sigma} \cdot \mathbf{E}_t. \quad (10)$$

Here, $\mathbf{J}_s$ is the surface current and $\mathbf{E}_t$ is the tangential electric field at the graphene plane. In the absence of a magnetic field, the dominant graphene plasmon is usually transverse magnetic (TM) polarized. Its electric field contains a longitudinal component along the propagation direction and a normal component perpendicular to the sheet. Once, $B_0 \neq 0$, the Hall conductivity couples TM and transverse electric (TE) field components. Consequently, the eigenmodes are generally hybridized and cannot be classified as purely TM or TE except in limiting cases [6].

A representative dispersion equation for a free-standing or dielectric-loaded graphene sheet contains both σ_{xx} and σ_{xy} . In schematic form, it may be written as:

$$D_{\text{TM}}(q, \omega, \sigma_{xx})D_{\text{TE}}(q, \omega, \sigma_{xx}) - \mathcal{C}(q, \omega)\sigma_{xy}^2 = 0, \quad (11)$$

Where, D_{TM} and D_{TE} are the uncoupled modal operators and $\mathcal{C}$ describes the strength of TM-TE coupling. The precise form depends on the dielectric geometry, retardation, and waveguide boundaries. The important point is that the Hall conductivity introduces a coupling term that vanishes at $B_0 = 0$. A magnetic field can therefore open spectral gaps, shift modal resonances, split degenerate modes, and transform the polarization state of the guided plasmon [26].

2.3 Bulk and Sheet Magnetoplasmons

Bulk or sheet magnetoplasmons are collective excitations distributed over the graphene plane rather than confined primarily to a physical edge. Their dispersion results from the interaction between the ordinary graphene plasmon and cyclotron motion. In the electrostatic approximation, a frequently used qualitative form is

$$\omega_{\text{mp}}(q, B_0) \approx \sqrt{\omega_p^2(q) + \omega_c^2} \quad (12)$$

where the zero-field graphene plasmon frequency follows approximately

$$\omega_p(q) \propto \sqrt{\frac{E_F q}{\epsilon}} \quad (13)$$

This expression is not universal; it neglects the full tensor boundary problem, nonlocality, dissipation, and retardation. Nevertheless, it captures the principal effect of magnetic bias: the plasmon resonance is shifted away from the zero-field dispersion and becomes dependent on both q and B_0 [7, 12].

At frequencies near the cyclotron resonance, the longitudinal and Hall conductivities vary rapidly. The resulting magnetoplasmon may exhibit strong absorption, enhanced polarization conversion, and reduced propagation length. At higher frequencies, the response can become less dissipative if the operating frequency lies outside the strongest interband and cyclotron-loss regions. The optimal regime for waveguiding therefore depends on the balance between magnetic tunability and carrier scattering. High mobility, low disorder, and careful control of the dielectric environment are essential for obtaining spectrally narrow and spatially extended modes.

In patterned graphene, geometric confinement modifies the bulk magnetoplasmon spectrum. Localized resonances are found in graphene disks, ribbons, nanoribbons and periodic arrays, with their frequencies depending both on the geometry and the carrier density. If the magnetic bias is present, each localized resonance can split into two individual branches corresponding to the opposite circular polarizations. The split will grow as magnetic field increases and is related to the chemical potential, damping rate, shape anisotropy and electromagnetic interaction between the adjacent elements. Theoretical works of graphene disks find that the chirality of the two magnetoplasmon branches can be different and, in some regimes, one branch can transform into a damping less edge localized mode [27].

In finite structures bulk and edge modes do not always have a well-defined separation. With reduced chemical potential, the magnetic

response increases in comparison to the ordinary plasmonic response, and the field distribution can be shifted toward the boundary. In this regime, the magnetoplasmon modes may become circulating and electric current will be preferentially circulated along the perimeter. The modes are of interest especially for small resonators, circulators and directional energy transfer devices where the distribution of current is related to the sign of the Hall conductivity.

2.4 Edge Magnetoplasmons and Chiral Propagation

The Edge magnetoplasmons occur due to the charge-density oscillation being localized at the edge of the two-dimensional conducting system. Under perpendicular magnetic field, the Hall current forces the charge to accumulate at the edge. This electrostatic restoring force and Hall drift are combined to produce a wave which propagates along the boundary. Contrary to the conventional plasmons, the propagation direction of edge magnetoplasmons is naturally dependent on the sign of the magnetic field, the type of charge carrier and the Hall conductivity [10, 13].

The chirality can be explained classically. Suppose you have an edge of graphene parallel to the y-axis and a magnetic field perpendicular to the plane. The electric field is perpendicular to the edge when the perturbation of the charge oscillates. This transverse electric field is made to flow parallel to the edge by means of the Hall conductivity. Reverse B_0 and the Hall current will be reversed, which will reverse the propagation direction. Likewise, electron to hole doping switch the sign of the Hall response and can switch the edge-mode chirality. Edge magnetoplasmons based on graphene are a unique form of electrical and magnetic control.

The dispersion of an edge magnetoplasmon is dependent on the electrostatic screening. The mode frequency is frequently a characteristic logarithmic function of the wave vector on an unscreened edge, written schematically as;

$$\omega_{\text{EMP}}(q) \sim \frac{\sigma_{xy}}{\epsilon_{\text{eff}}} q \ln \left(\frac{1}{|q|a} \right) \quad (14)$$

where a is an effective edge or screening length and ϵ_{eff} is the effective dielectric constant. A nearby metallic gate modifies the Coulomb interaction and can transform the dispersion toward a more nearly linear form. Gate screening is therefore an important design parameter in graphene magnetoplasmon waveguides. It controls the phase velocity, field confinement, interaction range, and sensitivity to remote disorder [23].

Experimental studies have demonstrated propagation of edge magnetoplasmons in graphene in the quantum Hall regime. These measurements revealed chiral propagation along the sample perimeter and indicated substantially longer relaxation times than those typically associated with nonchiral zero-field plasmons. The long-lived character results from the suppression of backscattering when only one propagation direction is available along a given edge. However, practical robustness depends on the quality of the edge, the filling factor, contact geometry, disorder, and the degree of coupling to bulk states. Edge roughness and inhomogeneous carrier density can still produce mode conversion and attenuation [11].

2.5 Nonreciprocity and Mode Hybridization

The most technologically important consequence of magnetization is the nonreciprocal nature of the resulting plasmon modes. For a reciprocal graphene waveguide, the forward and backward modes are related by time-reversal symmetry and have identical dispersion in magnitude. Under magnetic bias, the Hall conductivity breaks this symmetry. The dispersion can then satisfy

$$\omega(q, B_0) \neq \omega(-q, B_0) \quad (15)$$

Although the relation;

$$\omega(q, B_0) = \omega(-q, -B_0) \quad (16)$$

When the magnetic field is switched off with the propagation direction, the result is that usually remains. This asymmetry can be in the form of unequal phase constants, attenuation rates, field profiles, or coupling efficiencies, for the two directions.

The magnetic field can hybridize the graphene plasmon with dielectric waveguide modes in a dielectric-loaded graphene waveguide.

Consequently, instead of a single isolated plasmon branch, it is a family of magnetoplasmon-polaritons. A relatively weak magnetic field can create a gap in the frequencies for one of the lowest-order modes; a larger magnetic field will affect the cutoff frequencies and the ratio of TM and TE fields. Propagator control without changing physical waveguide geometry is important, and this hybridization is the means [12].

The mode conversion at TM-TE mode changes is more conspicuous near the anticrossings. These are the locations at which the eigenmodes can swap their confinement, group velocity and loss properties. Avoided crossings can thus be utilized in the design of tunable filters, mode converters, and switches. The ratio of Hall conductivity to longitudinal conductivity, the dielectric asymmetry above and below graphene, and the overlap between the uncoupled modes, all control the operating behavior.

2.6 Quantum Magnetoplasmons

At sufficiently strong magnetic fields or low carrier densities, the classical description becomes inadequate because graphene's electronic spectrum is quantized into Landau levels. For monolayer graphene, the Landau-level energies are approximately:

$$E_n = \text{sgn}(n)v_F\sqrt{2e\hbar B_0|n|} \quad (17)$$

where $n = 0, \pm 1, \pm 2, \dots$. Optical transitions between Landau levels generate resonances in the longitudinal and Hall conductivities. Magnetoplasmons in this regime are therefore influenced by the filling factor, transition energies, level broadening, and occupation probabilities [1, 25].

Alongside semiclassical collective modes, quantum magnetoplasmons can exist. The Landau-level spacing becomes comparable to the thermal and disorder broadening at high filling factors and the response becomes close to the classical Drude response. Discrete transitions dominate at low filling factors, and can significantly alter the plasmon dispersion. Couplings between collective charge oscillations and inter-Landau level transitions can lead to

further avoided crossings, gaps and long-lived edge states. They are relevant for quantum-Hall plasmonics, THz spectroscopy and for quantum-compatible nonreciprocal devices.

Thus, the difference between classical and quantum magnetoplasmons is not only devised by the size of B_0 , but also by the relative size of Landau-level spacing, temperature, disorder broadening, Fermi energy, plasmon energy and the carrier relaxation rate. A complete theory may require a nonlocal Kubo conductivity, self-consistent treatment of electrostatic screening, and realistic modeling of the edge potential [28].

3. GRAPHENE CIRCULAR WAVEGUIDES

Graphene circular waveguides are cylindrical waveguides having a monolayer of graphene on the surface of a dielectric nanowire or between concentric dielectric regions. Cylindrical core of radius R , permittivity of the core ϵ_{in} , permittivity of the surrounding ϵ_{out} , graphene sheet at $r = R$; Electromagnetic fields propagating along the cylinder axis:

$$\mathbf{E}, \mathbf{H} \propto \exp(i\beta z + im\phi - i\omega t) \quad (18)$$

Here, the complex propagation constant is β , and the azimuthal mode number is m . The real part of β regulates the propagation of the phase and the imaginary part represents the attenuation due to the absorption and scattering of carriers within graphene and radiation leakage and the loss of carriers from the substrate. Unlike planar graphene ribbons, circular waveguides have no edges in the direction of propagation, which helps to minimize edge scattering and give a smooth transition for highly localized plasmon propagation [29, 30].

Graphene can be viewed as a very thin conductive border in general. Its surface current is related to the tangential electric field as given in equation (2). In the terahertz and mid-infrared regimes, the intraband contribution is generally more important, and can be modeled by a Drude-type formula:

$$\sigma_{intra}(\omega) \approx \frac{ie^2 E_F}{\pi\hbar^2(\omega + \frac{i}{\tau})} \quad (19)$$

The chemical potential E_F and the carrier relaxation time τ . The interband contributions

are important when $\hbar\omega$ is not too far from $2E_F$. Therefore, the interband terms and intraband terms should be considered in an accurate model particularly for the near-infrared and visible operations [30, 31].

The field modes inside the dielectric core are Bessel or modified Bessel functions which are finite at the cylinder axis, and the field modes outside of the graphene layer are decaying functions. One can arrive at a characteristic equation of: using continuity of the tangential electric field and the magnetic field as a discontinuity, caused by the surface current of the graphene.

$$D_m(\beta, \omega, R, \varepsilon_{in}, \varepsilon_{out}, \sigma_g) = 0 \quad (20)$$

The solutions of this equation are the complex propagation constants of the guided modes. In general, the fundamental mode is TM-like mode, and has significant radial and longitudinal electric-field components near the graphene surface. The majority of modes are of hybrid (TE-TM) nature because the fields are combined in the longitudinal direction and in the azimuthal direction, respectively, due to cylindrical symmetry. Normally the azimuthally symmetric fundamental modes are only classified as purely TE or TM [29].

The characteristics of the circular graphene waveguides are the deep-subwavelength confinement and electrical tunability. The effective mode number is:

$$n_{eff} = \frac{\text{Re}(\beta)}{k_0} \quad (21)$$

Where, $k_0 = \omega/c$. The propagation length can be expressed as:

$$L_p = \frac{1}{2\text{Im}(\beta)} \quad (22)$$

Although definitions based on amplitude decay are also common, when intensity decay is used. The effective mode area comes from the distribution of the electromagnetic energy, and is typically normalized to the diffraction-limited area. The localization/loss tradeoff is one of the most central design problems in that the higher the localization, the more they overlap with graphene, but the more dissipative is the absorption [30].

3.1 Effects of Radius, Chemical Potential, and Magnetic-field

3.1.1 Influence of Radius

The radius R has an important influence on the dispersion, mode order, localization and cutoff of the fields. The cylindrical interface can be simplified to a planar graphene-dielectric interface if R is much larger than the plasmon decay length. Smaller radii change the field distribution in the radial direction, and change the ratio of the longitudinal to azimuthal momentum. The suppression of the higher order modes, by reducing the radius, usually leads to single mode and the fundamental mode is often cutoff free [30, 32].

A smaller radius also results in the physical dimensions being smaller, and may lead to better subwavelength confinement. However, strong curvature results in sensitivity to the surface roughness, the nonlocal conductivity, quantum confinement, and departure from the local surface-conductivity model. The classical description then needs to be compared with the nonlocal or quantum description of the case of very small radii. Typically the radius of the cylindrical waveguide supports a larger number of azimuthal modes, roughly in proportion to the circumference being able to accommodate more modal wavelengths. It can also cause bigger modes and to a smaller effect of curvature.

TE-like modes differ from TM plasmons in the way that the radius of the modes depends. A negative imaginary part of the conductivity is required for the formation of TE-mode in a cylindrical graphene waveguide, but not enough. The cylinder should also have a radius R larger than a certain value that is dependent on frequency and the permittivity contrast between the core and the medium. This critical radius can be similar to the free space wavelength for typical dielectric settings. The best condition is achieved when the permittivities of the core and the outer media are the same. The effective conductivity can be enhanced by multilayer graphene, thus decreasing the critical radius [29].

3.1.2 Influence of Chemical Potential

The electrical control of guided modes is direct and is achieved by the chemical potential. In the intraband regime, the graphene-plasmon frequency is more or less proportional to equation (12). At constant frequency, the plasmon wave number decreases as E_F increases. At constant frequency, the effective index decreases as E_F increases. The mode propagation is not as localized, the propagation length is typically longer because of the amplification of the non-dissipative part of the propagation of the mode, but the dissipative part of the propagation is suppressed by the suppression of interband absorption [1, 32].

Thus, lower the chemical potential, the more confinement in the field and the longer the propagation, the higher the effective index. It has been reported that the mode index of coated nanowire can be reduced and the propagation length can be increased by increasing the chemical potential in graphene-coated nanowires and related structures. This is an important exchange for device design. Near field enhancement is more desirable for sensors and nonlinear optical devices, but is not desirable for communication and signal routing applications [1].

Also, it is the chemical potential which determines the spectral range of existence of a mode. Interband transitions are Pauli blocked and TM plasmons are usually dominant for $\hbar\omega < 2E_F$. For interband transitions, near $\hbar\omega \approx 2E_F$, the conductivity can be significantly changed and TE-like modes can be made possible. Therefore, it is possible to switch between different modal regimes or modify higher order mode cutoff conditions and switch a waveguide to reconfigurable single mode operation using electrostatic gating [29].

3.1.3 Influence of Magnetic-field

If a magnetic field is applied perpendicular to the plane of the graphene, then the conductivity of the graphene will be represented by equation (2) and it will be gyrotropic. The Hall term is a combination of both radial and azimuthal current in cylindrical coordinates. Thus the

modes that are degenerate at zero fields might split in the direction of the circulations. Thus, different propagation constants can correspond to the modes of azimuthal index $+m$ and $-m$, respectively, which has the clockwise and counterclockwise propagation of energy or current, respectively; and these modes can be independent.

$$\beta_{+m}(B) \neq \beta_{-m}(B) \quad (23)$$

A change of the magnetic field changes the sign of the Hall conductivity, and interchanges the two branches of the mode. The opposite effect can be achieved by changing the carriers' charge and Hall response from electron to hole doped by switching the carrier type [2, 25].

Further, hybrid modes are hybridized modes of TE-TM. In the radial electric field, an azimuthal Hall current will be formed, and the azimuthal electric field will form a radial current. The eigenmodes that are produced are generally hybrid and can be elliptically polarized, rotating or nonreciprocal near fields. In a simplified semiclassical treatment the relation of the magnetoplasmon frequency is shown to be close to that of the upper hybrid frequency (equation 6), with the graphene cyclotron frequency (equation 3). The relation between the magnetic tuning and electrostatic gating (higher the B stronger the cyclotron contribution and higher the E_F lower the ω_c for a given B) can be useful in controlling mode splitting and directionality away from cyclotron resonance, or Landau-level resonances; and can be useful in enhancing absorption near these resonances [2, 25].

4. ANALYTICAL AND NUMERICAL MODELING

The cylindrical coordinate separation of Maxwell's equations makes the use of analytical models particularly useful for circular waveguides. Dispersion relations and cutoff conditions and even approximate scaling laws can be obtained from Bessel-function expansions, impedance-boundary conditions, and determinant-based eigenvalue equations. These methods are computationally efficient and provide physical

understanding of the effect of radius, dielectric contrast, chemical potential, and magnetic bias [32, 33].

Transfer-matrix and Mode Matching Methods are helpful for multi-layer and coaxial structures. A field-expansion matrix is used to describe each cylindrical region and the continuity conditions are imposed at the graphene/dielectric interfaces. The tensorial conductivity can be directly used to calculate the magnetically induced mode splitting and hybridization.

For complex geometries, finite substrates, nonuniform gating, patterned graphene and strongly hybridized modes, full wave numerical methods are necessary. The finite-element method is a popular method for calculating complex Eigen frequencies or propagation constants, field distribution, propagation length and effective mode area. Bends, couplers, pulsed excitation, and dynamically modulated magnetic fields are examples of finite-length waveguides in which finite-difference frequency-domain and time-domain methods are useful. Another advantage of using the BEM (boundary-element methods) is that the surrounding volume does not need to be meshed since graphene is naturally described as a surface current [1, 30].

All of the above should be specified in reliable simulations: conductivity model, temperature, relaxation time, chemical potential, orientation of magnetic field, and dielectric losses. Convergence tests are crucial as plasmon wavelength can be very small compared to the free space wavelength. In the limit cases ($B \rightarrow 0$, $R \rightarrow \infty$, and $\sigma_H \rightarrow 0$) analytical and numerical solutions should also be compared. This type of validation is especially crucial to differentiate true from false nonreciprocity from purely numerical errors.

Graphene circular waveguides are thus an ideal platform for a compact and reconfigurable strong confinement of the plasmon transport. They exhibit properties which can be tuned by radius, chemical potential, dielectric environment and magnetic field. The cylindrical confinement and magnetic control of the Hall conductivity is particularly promising for nonreciprocal

waveguides, tunable resonators, mode conversion, and integrated plasmonic circuits.

4.1 Analytical and Numerical Methods

Analytical and numerical tools are required for theoretical study of plasmonic modes in graphene-embedded and plasma-filled waveguides, which must be applicable for the dispersion of the material, high field localization, complex interfaces and, in magnetized waveguides, the tensor electromagnetic response. The selection method influences the accuracy of the prediction of the propagation constant, the effective modal index, attenuation, cutoff conditions, dispersion relation and electromagnetic field distribution. Another model problem that arises in graphene structures is that of the very small thickness of the material. The easiest way to visualize graphene is as an ultrathin volumetric layer with an equivalent permittivity, while for many full-wave simulations it is more convenient to consider graphene as a surface-current boundary (equation (10)). The surface-conductivity formulation eliminates the need for artificially assigning a thickness to graphene and, in general, decreases the meshing problems in finite-element simulations [1].

4.1.1 Effective Dielectric Constant Method

For structures with multiple guiding regions of different dielectrics or for confined guiding regions the effective dielectric constant method (EDCM) (also called effective-index method in related waveguide studies) is a relatively easy analytical approximation. The concept is to find a region, equivalent to the complicated cross section in space, but having a "homogeneous" refractive index or dielectric constant. Original multidimensional modal problem can then be reduced into a series of one dimensional problem. In a dielectric-loaded graphene plasmon waveguide, for instance, the effective index of the dielectric-loaded portion can be first calculated and then the propagation constant of the graphene-supported plasmon can be calculated. This method works well if the width (transverse scale) and length (longitudinal scale) of the field are sufficiently large, the field changes slowly

along the width, and there are no strong discontinuities in the structure along the length [2].

The major benefit of EDCM is that it is low cost computationally. It allows for fast sweeps of the parameters, including frequency, graphene chemical potential, dielectric thickness, waveguide width, and carrier relaxation time. Therefore it is useful during the initial design phase when approximate values of propagation length, cutoff frequencies of the modes, approximate confinement trends etc. are required. The studies of terahertz dielectric-loaded graphene waveguides have demonstrated that the principal modal properties of the full finite-element model could be accurately reproduced by the effective-index approximation, which includes the effective mode index, the modal loss, and the higher-order-mode cutoff behavior. The agreement is particularly advantageous if the field is well concentrated around the graphene layer and the loading of the dielectric can be considered as a perturbation of the underlying graphene plasmon [2].

4.1.2 The Plane-Wave Expansion Method

The most popular spectral method that is used for periodic dielectric, plasmonic and metamaterial structures is the plane-wave expansion method (PWEM). In PWEM, the spatially varying permittivity and/or conductivity is expanded in Fourier components, and the fields are expressed as a superposition of plane waves. The distribution of the material is periodic for:

$$\epsilon(\mathbf{r}) = \sum_{\mathbf{G}} \epsilon_{\mathbf{G}} e^{i\mathbf{G} \cdot \mathbf{r}} \quad (24)$$

where $\mathbf{G}$ denotes reciprocal-lattice vectors. Applying Bloch's theorem and substituting the Fourier expansions into Maxwell's equations produces a matrix eigenvalue problem. Solving this problem for a sequence of Bloch wave vectors yields the photonic or plasmonic band structure, group velocity, bandgap, and mode profile [29].

Periodic graphene metasurfaces, patterned graphene ribbons, plasmonic crystals, graphene-dielectric superlattices and arrays of plasma-filled

inclusions are all examples of structures that can be particularly beneficial for PWEM. It has the natural incorporation of periodic boundary conditions via Bloch's Theorem and a direct interpretation of the band formation to be the effect of the coupling of Fourier harmonics. The technique can also be applied to the study of Bragg scattering, flat bands, band folding, symmetry-protected crossings, topological and nonreciprocal band features. The Hall response can be incorporated into the Fourier domain eigenvalue problem and the effect on band splitting and directionality of propagation studied in magnetized graphene.

However, PWEM has its drawbacks for strong dispersive plasmonic media. Graphene's conductivity is frequency-dependent, chemical potential-dependent, temperature-dependent, relaxation time-dependent, and magnetic field-dependent. The longitudinal and Hall components of the magnetized system are dispersive. Hence, it is not enough to use a conventional frequency independent eigenvalue formulation unless the material response is included as a nonlinear eigenvalue problem, implementing auxiliary fields, fitting the material response with a rational function, or performing iterative frequency domain methods. Furthermore, there can be a discontinuity in conductivity or a sharp discontinuity in material properties which can result in a high number of Fourier harmonics needed to converge. Truncation may lead to Gibbs-type errors and unphysical field values at graphene interfaces. However, idealized infinite periodic systems are the natural setting for PWEM, less natural for finite waveguides, less natural for isolated defects, less natural for nonperiodic bends and tapering, and less natural for open structures with complex radiation conditions [29].

4.1.3 The Finite Element Method

Finite element method (FEM) is one of the most popular numerical methods used for the graphene plasmonic waveguides, which offers more geometrical flexibility. The computational domain in FEM is subdivided into small elements and local basis functions are used to approximate

the electromagnetic fields. The weak form of Maxwell's equations is then solved as an eigenvalue problem, a frequency domain problem, or a driven scattering problem. FEM is applicable for arbitrary cross sections, curved interfaces, multilayer structures, anisotropic materials, varying magnetizations, varying chemical potential, finite substrates and nonuniform magnetic fields. It is therefore well suited to circular graphene waveguides, graphene-coated nanowires, dielectric-loaded graphene waveguides, hybrid graphene-metal waveguides and plasma-filled cylindrical waveguides [1, 34].

Graphene can be incorporated in a FEM using a surface-current boundary condition, a transition-boundary condition or an equivalent thin volumetric layer. The surface formulation is normally the preferred option, as the introduction of an artificial thickness to graphene can generate very stringent mesh requirements and numerical instability. The equivalent permittivity of graphene, a model of a layer of thickness: t_g , is frequently expressed as:

$$\varepsilon_g(\omega) = 1 + \frac{i\sigma(\omega)}{\omega\varepsilon_0 t_g} \quad (25)$$

However, the result would be sensitive to the value of t_g chosen, especially if the mesh is not sufficiently resolved to cover the thickness of the layer. In a surface-conductivity model, no such ambiguity exists and allows to use anisotropic conductivity tensors for magnetized graphene [1]. The computation of complex propagation constant β , effective index, propagation length, electromagnetic energy density, mode area, field enhancement, and modal overlap, is particularly useful for FEM. It can also solve modes that don't have an easy TE or TM description such as hybrid modes. In a magnetized circular graphene waveguide, for instance, the Hall conductivity has a coupling between radial and azimuthal current components, resulting in hybrid modes of currents with longitudinal and transverse field components. This type of coupling is hard to model with a scalar effective-medium approximation, but can be introduced directly into a full-vector FEM formulation.

Other numerical methods can be used in addition to EDCM, PWEM and FEM. The transfer-matrix method is efficient for multilayer planar or cylindrical structures and is applicable to calculation of dispersion for multiple interfaces. The boundary element methods discretize only surfaces and are appealing for particles coated with graphene, nanowires and interfaces. The finite-difference frequency-domain and finite-difference time-domain methods are applicable for finite devices, transient excitation, bends, couplers, and dynamically modulated graphene structures, but the short plasmon wavelength may create a challenging spatial and temporal resolution requirement [35].

Therefore, the type of modeling technique chosen should be dependent on the geometry, material response, periodicity and desired output. EDCM is beneficial in situations where analytical estimates and studies of parameters occur quickly in relatively simple or weakly coupled structures. PWEM is also suitable for infinite periodic systems and calculations of the band structure as long as the graphene dispersion and plasma dispersion are carefully included. The major advantage of using FEM is the ability to model complicated geometries, curved interfaces, anisotropic or magnetized media, and detailed field calculations, although it typically requires more computational resources. These approaches are usually linked in a comprehensive investigation; an initial design estimate by EDCM, periodic-band effects by PWEM, and full electromagnetic fields by FEM. This multilevel approach is particularly useful for magnetized graphene waveguides, where the curvature, material dispersion, Hall conductivity, mode hybridization, and losses must all be taken into account. The key analytical and numerical methods for the investigation of plasmonic modes in graphene-based and plasma-filled waveguide structures are summarized in **Tab. 1**, where the advantages and the limitations of the methods are also indicated, as well as the typical applications.

Table 1: Comparison of analytical and numerical methods for investigating graphene plasmonic waveguides

Sr.	Technique	Principle	Main Parameters/Outputs	Ref.
1	Finite element method (FEM) for waveguide eigenmodes. Nonlinear properties extracted via nonlinear Schrödinger equation.	Theoretical and numerical modeling of wave propagation. Focus on linear and nonlinear regimes in graphene.	Waveguide eigenmodes calculated using FEM. Nonlinear properties extracted via nonlinear Schrödinger equation framework.	[36]
2	Three-dimensional FEM formulation. Solves electromagnetic problems with electric and magnetic fields.	Solves electromagnetic problems using electric field and magnetic flux. Models graphene as a finite conductivity boundary condition.	Effective refractive index as unknown eigenvalue. Accurate dispersion diagrams and field distributions.	[37]
3	2-Dimensional (2-D FEM) formulation. Modal computation for waveguides with graphene sheets.	2-D FEM for modal computation in waveguides. Models tensorial conductivity with mixed elements.	Modal computation of waveguide properties. Comparison of results with different techniques.	[38]
4	FEM solver developed. Augmented eigenvalue problem for graphene representation.	Developed FEM solver for graphene's quasinormal modes. Analyzes graphene's conductivity and magneto-optical properties.	Quasinormal modes (QNMs) of graphene analyzed. Absorption cross section (ACS) validated against commercial solver.	[39]
5	Mixed finite-element method with impedance transmission boundary condition. Models field penetration through thin graphene sheets.	Mixed FEM combines Gauss' law and Helmholtz equation. ITBC (impedance transmission boundary condition) models field penetration through graphene sheets.	Fermi levels: 0.2 eV, 0.7 eV. Spacer thicknesses: 2 nm, 4 nm.	[40]
6	FEM and COMSOL software used. Analyzed modal characteristics of waveguide.	Utilizes graphene and silicon nanowires for waveguiding. Enhances field confinement and propagation distances.	Strong field confinement: $\lambda^2/100$ to $\lambda^2/17$. Propagation distances: 86-215 μm .	[41]
7	Analytical model using Floquet-Bloch theory (FBTH). Models spatiotemporal modulation in graphene structures.	FBTH periodic electromagnetic responses. Graphene's surface current imposes boundary conditions on modes.	Mode conversion and wave amplification phenomena analyzed. Interband and intraband transitions between modes investigated.	[42]
8	Method of lines based on Maxwell's equations. Simulation and analysis of plasmonic waveguide	Electric field alters graphene's Fermi level. Modulation based on effective index and absorption.	Fermi level affects modulator performance and design. Minimum device length	[43]

Sr.	Technique	Principle	Main Parameters/Outputs	Ref.
9	modulator. Transfer matrix theory for analytical analysis. Finite-element method for numerical analysis.	Transfer matrix theory and finite-element method used. Analyzes guided mode propagation in waveguides.	achieved is six nanometers. Propagation length, spatial length, electromagnetic field profile. Comparison of graphene and noble metals' performance.	[44]
10	Full-vectorial finite element formulation for analysis. Considers graphene as surface conductivity.	Full-vectorial formulation for eigenmode analysis. Graphene modeled as surface conductivity.	Full-vectorial eigenmode analysis of plasmonic waveguides. Graphene as surface conductivity enhances waveguide characteristics.	[45]
11	Theoretical and numerical investigation of waveguide. Analytic approach using electromagnetic theory.	Nonreciprocal behavior from ferrite's magneto-optical properties. Surface plasmon propagation tailored by external fields.	Propagation lengths tailored by magnetic bias fields. Nonreciprocal behavior near Larmor frequency observed.	[46]
12	Finite-difference time-domain (FDTD) method used. Evaluated Kretschmann and Otto configurations.	Surface plasmons enhance electromagnetic field intensity. Optimal metal/dielectric-graphene width maximizes magnetic field amplitude.	Optimal metal or dielectric-graphene width identified. Maximum magnetic field amplitude achieved in configurations.	[47]
13	Nonconventional discretization for FDTD method. Evaluates x and y-components at same location.	Nonconventional discretization for electromagnetic wave propagation. Evaluates x and y-components at the same location.	New band gaps and dispersion-less flat bands observed. Band locations controlled by magnetic field and plasma parameters.	[48]
14	FEM for modal analysis. Finite Difference Time Domain for dispersion estimation.	Surface plasmon modes propagate at metal-dielectric interfaces. Electromagnetic field interacts with free electrons in metals.	Waveguide width and spot size parameters analyzed. Dispersion characteristics and propagation loss evaluated.	[49]
15	FDTD and ADI (alternating-direction-implicit)-FDTD methods are reviewed. Coupled with hydrodynamic equations for modeling.	Coupling electromagnetic waves with electronic transport phenomena. Modeling requires solving electrodynamics' equations efficiently.	Electromagnetic wave coupling to devices. Speed of electrodynamics solution affected by grid sizes.	[50]

5. APPLICATIONS AND TECHNOLOGICAL RELEVANCE

The ability to confine the field to the extreme and to tune it actively makes graphene very appealing for integrated devices, where the optical function must fit into a small footprint. The tunability is due to the fact that the complex surface conductivity of graphene is a function of the chemical potential. The intraband-dominated regime shows approximately Drude-like conductivity that is strongly dependent on the Fermi energy, carrier relaxation time, temperature and frequency. The chemical potential is increased, generally, leading to higher graphene Drude weight and a shift of the dispersion of graphene plasmon, so that the electrical control of the modal index, plasmon wavelength, confinement, and propagation length are achieved. Experimental near-field studies showed that the propagating graphene plasmon wavelength and amplitude can be reconfigured by tunable gate voltage, which proves that graphene is not a passive plasmonic material, but an actively reconfigurable optical medium [22, 51].

The localization of graphene plasmons is strong, which significantly increases the electromagnetic field close to the graphene surface. This results in higher analyte molecule interactions with the surrounding dielectrics, phonons, quantum emitters and other 2D materials. Thus, graphene plasmonics is relevant to refractive-index sensing, biochemical detection, infrared spectroscopy and surface-enhanced light-matter interactions. One of the most crucial regions of the spectrum are the mid-infrared and THz regions where a number of molecular and biological excitations have characteristic vibrational or rotational fingerprints [52]. These long wavelengths can be compressed down to nanoscale volumes by graphene plasmons, which raise the overlap between the EM field and the target material. They can also be tuned by electrostatic gating and chemical doping to provide tunable instead of fixed frequency sensing platforms [32].

Graphene plasmonic sensors can be made in the form of nanoribbons, disks, patterned

metasurfaces, waveguides, resonant cavities or a hybrid graphene-dielectric structure. The plasmon dispersion or resonance frequency is then changed by change in external refractive index while the gate voltage constitutes an independent tuning mechanism to shift the operating point. This dual control can help to avoid misinterpretation of changes in the environment and can be used with differential sensing schemes in the fabrication. Moreover, the atomic thinness of graphene may bring the plasmonic field close to the analyte surface, thereby enhancing the surface sensitivity. Practical performance of sensors, however, continues to rely on the mobility, disorder, phonons in the substrate, adsorption of analyte, the resonance linewidth, and the coupling between free space radiation and the extremely localized plasmon mode [22].

Graphene plasmons are also potential for photodetectors. Graphene can absorb incident radiation via intraband transitions at THz and mid-infrared frequencies, and local enhancements of the optical field in a small active area due to plasmonic confinement can boost absorption. The plasmon-assisted detection mechanisms are enhanced photothermoelectric response, bolometric effect, photovoltaic response, rectification of plasma oscillations and photoconductive processes. Graphene plasmonic resonances can focus the incident radiation into the conducting sheet to enhance the interaction between the optical field and mobile carriers. The plasmon-enhanced intraband absorption and charge-density-wave rectification are pointed out as promising approaches for developing compact photodetectors in graphene THz devices. Also, the tunability of graphene is electrical, enabling a detector to be tuned into a desired spectral band without the replacement of active material or mechanical changes to the device [1].

The wave propagation properties are the same for electro-optic and optoelectronic modulation. A graphene modulator can be operated through changing the carrier concentration and thus altering a plasmonic resonance, altering the absorption, or altering the coupling between a

graphene mode and a dielectric or photonic mode. At a resonance, quite small changes in conductivity can result in a significant change of transmission, reflection, phase, or polarization. Ribbons, disks, resonant cavities, dielectric-loaded waveguides and hybrid silicon-graphene photonic structures can thus be used for implementing graphene-based modulators. It can be carried out using absorptive, dispersive or electrically controlled mode coupling modulation mechanisms. Graphene is atomically thin, allowing it to be used in integration with current photonic platforms, with minimal extra volume and without the need for a thick active semiconductor layer [12].

Graphene plasmonic waveguides could offer a possible path to making optical information-processing systems smaller. In conventional dielectric waveguides, diffraction limits the dimensions and they usually require dimensions of the order of the free space wavelength. By providing the ability to guide electromagnetic energy at much shorter wavelengths, graphene plasmons enable the reduction of bends, splitters, resonators, phase shifters and modulators. Another advantage of the graphene-based waveguides is the possibility to reconfigure the path of the plasmonic signals dynamically by means of an electrical control. The compactness and tunability promise dense on-chip interconnects and reconfigurable optoelectronic circuits in principle [12, 29].

The higher the degree of confinement, the higher will be the degree of absorption [53]. The closer the plasmon energy is to graphene, the greater the fraction of energy stored close to graphene, and the greater the loss from other mechanisms, such as scattering from carriers, attenuates the plasmon. Propagation length is related to the mobility, impurity scattering, optical phonons, and interactions with substrates, temperature and frequency of operation. High quality hexagonal boron nitride may be effective in reducing disorder and enhance propagation of plasmon modes, but might also lead to extra resonances and loss channels due to graphene-phonon-polariton hybridization. Therefore, there is a

trade-off between confinement of modes, propagation length, coupling efficiency, and fabrication complexity in the device design [26].

Another degree of geometrical control is given by circular graphene waveguides. For circular or cylindrical geometry, the graphene layer is in a continuous interface around a core of a dielectric, allowing for both longitudinal propagation and azimuthal angular momentum. The radius defines the curvature, modal confinement, cutoff and the azimuthal modes which are supported. Smaller radii can lead to small and even a single-mode device, while larger radii minimize the curvature effects and approach the behavior of a planar graphene interface. The cylindrical geometry is also beneficial for nanowires, coaxial structures, resonators, and fiber compatible devices [12].

The combination of magnetized plasma, magneto-optical dielectric media and graphene can further modify the dispersion relation. Hybrid magnetoplasmonic modes are created by the interaction between the graphene plasmon and the cyclotron motion in the magnetized medium. The frequency splitting, avoided crossing, field confinement and asymmetric propagation of the resulting modes may depend on the magnetic field, carrier density, plasma frequency and geometry. The coupling between the cylindrical angular momentum and the Hall response could be especially helpful in controlling the circulation and polarization of modes in circular waveguides. These structures could be a potential solution to enhance directional transmission while keeping the compact size of graphene plasmonics devices.

6. CHALLENGES AND FUTURE PROSPECTS

There are, however, several present issues to be addressed before the application of circular or magnetized waveguides becomes feasible with graphene plasmons, such as the high confinement and electrical tunability of the plasmons. Propagation loss due to carrier-phonon scattering, impurity disorder, interactions with the substrate, edge roughness, and the quality of

graphene is the most critical limitation [32]. These mechanisms increase the plasmon resonance and shorten the propagation length, especially for the strongly-proximal mode. While encapsulation of graphene with hBN can vastly enhance mobility and minimize the disorder of the substrate, it is still somewhat complicated and difficult to scale up to large area production of high-quality hBN-graphene-hBN stacks. Another big challenge arises due to the large momentum mismatch between free space and highly compressed graphene plasmons [1]. Excitation can be enhanced with the use of an antenna, a grating, a prism coupler, and a tapered waveguide, but these results in extra fabrication requirements and can also result in increased insertion loss. A strong, uniform magnetic field is necessary in magnetized structures, which adds another challenge for integration. System size and power consumption can be increased by external magnets, while losses dependent on magnetic field, Landau-level transitions, nonlocal conductivity, thermal effects and chemical potential variations across the system result in some difficulties for accurate modeling. In addition, the curvature, surface roughness, azimuthal-mode splitting and possible TE-TM hybridization considerations must be taken into account in designing reliable devices for the circular waveguide [2].

Advanced materials, advanced geometries and efficient electrical or magnetic control will all be needed in the future. A higher-mobility graphene, cleaner interfaces, hBN encapsulation, multilayer graphene, and carefully-designed dielectric environments could enhance propagation length while maintaining strong confinement [32]. Improved coupling efficiency may be achieved with hybrid plasmonic architectures of graphene with dielectric waveguides, photonic crystals, metasurfaces, phonon-polaritonic materials, or low-loss resonators, which can break the coupling efficiency/confinement/attenuation balance. The Hall conductivity provides a means to nonreciprocal phase control, directional mode conversion, tunable circulators, isolators and chiral or topologically robust edge channels in

magnetized circular waveguides. Future devices could use both electrostatic gating and magnetic bias to manipulate the frequency, polarization, propagation direction and mode order in the same structure. If these concepts are to be realized in practical THz and mid-infrared communication systems, compact sensors, low-power gate control and high-speed optoelectronic platforms, they will require advances in wafer-scale fabrication, nanoscale patterning, integrated magnetic materials, and low-power gate control [1, 15, 32]. The performance of hybrid nanophotonic structures should also be optimized by rigorous control of the dispersion of materials, the bonding of interfaces, particle distribution, and processing parameters. The dispersion of the magnetic nanoparticles, the engineering of the interfaces, magnetic alignment and the scalability of the processing can have a significant impact on the functional performance of magnetic nanoparticle-polymer nanocomposites, and similar considerations should be taken into account when designing hybrid graphene-based magnetoplasmonic architectures [54].

7. CONCLUSION

The combination of the tunability of the graphene Fermi energy and the magnet-induced gyrotropy opens up the prospect of controlling highly-confined plasmonic modes on magnetized graphene waveguides. The Hall conductivity induced due to an external magnetic field can be exploited for mode splitting, hybridization, asymmetric propagation and chiral edge states, which are additional parameters of control beyond the conventional graphene plasmons. The geometry of circular graphene waveguides is especially appealing because it allows for strong sub-wavelength confinement and enables changes in mode properties and propagation via the radius, chemical potential, dielectric environment and magnetic field. These can be analyzed using analytical and numerical methods such as EDCM, PWEM, FEM, transfer-matrix, boundary-element or time-domain methods; the full-vector FEM approach has proved to be a useful tool for complex curved structures and/or anisotropic

configurations. Even with these benefits, these practical limits are propagation losses, disorder, imperfect coupling, curvature effects, nonlocal response, and the difficulties to integrate strong magnetic bias. Improvements in graphene quality, low loss interfaces, optimized hybrid architectures, and electrical and magnetic control simultaneously will allow for compact nonreciprocal waveguides, circulators, isolators, switches, sensors, and other tunable photonic devices to become a reality.

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